Complexity Theory B

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`You are encouraged to work together on solving homework problems, but please put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class.`

Homework 1, due April 29

1. An oracle Turing machine is *unary* if all queries it makes to the oracle are of the form $11\ldots1$.

   Prove that the class of languages that can be recognized by a polynomial time unary oracle machine with any oracle is equal to $P/poly$.

2. Prove that $\mathrm{MOD}_{2010}(x_1, \ldots, x_n)$ is computable by a bounded depth polynomial size circuit in the basis $\{\neg, \mathrm{MAJ}\}$.

3. Prove Spira's theorem for the monotone basis $\{\wedge, \vee\}$. That is, prove that for any monotone formula F of size s there exists an equivalent formula of depth $O(\log s)$.

4. Prove that the problem of testing if a given undirected graph is bipartite (that is, 2-colorable) belongs to the class L.

   `Hint.` You may use Reingold's result $SL = L$.

5. Recall that *the formula size* $L(f)$ of f is measured by the number of computational gates in a minimal $\{\neg, \wedge, \vee\}$-formula computing f. Prove that whenever

$f_1(x_{11}, \dots, x_{1n_1}), \dots, f_m(x_{m1}, \dots, x_{mn_m})$ are arbitrary Boolean functions with pairwise disjoint sets of variables, we have the inequality

$$L(f_1(x_{11}, \dots, x_{1n_1}) \oplus \dots \oplus f_m(x_{m1}, \dots, x_{mn_m})) \geq \frac{1}{2} \sum_{i=1}^{m} L(f_i).$$