Complexity Theory B

Instructor: Alexander Razborov, University of Chicago
razborov@cs.uchicago.edu
Course Homepage: www.cs.uchicago.edu/~razborov/teaching/spring10.html

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`You are encouraged to work together on solving homework problems, but please put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class.`

Homework 2, due May 20

1. Prove that $\mathrm{Corr}(\mathrm{PARITY}_n, \mathrm{MAJ}_n) = \Theta(n^{-1/2})$.

   `Note.` You need to prove *both* upper and lower bounds.

2. A branching program is *leveled* if its nodes can be arranged as $\{\, v_{ij} \mid 1 \le i \le t, 1 \le j \le w \,\}$ in such a way that all wires have the form $(v_{ij}, v_{i+1,j'})$. Let us call a leveled program *one-way* if in addition every wire satisfies $j' \ge j$.

   Prove exponential lower bounds for the parity function in the model of **bounded-width** one-way branching programs.

3. Prove that $NC(\mathrm{MAJ}_n(x_1 \wedge y_1, \ldots, x_n \wedge y_n)) \ge \Omega(n)$ (NC is the non-deterministic communication complexity, Alice gets $x_1, \ldots, x_n$, Bob gets $y_1 \ldots, y_n$).

4. For a function $f : X \times Y \longrightarrow \{0,1\}$, $\chi_1(f)$ is the minimal number of **disjoint** 1-rectangles covering $f^{-1}(1)$.

   Prove that $C(f) \le O(\log \chi_1(f)^2)$.

5. Let $A(x_1, \ldots, x_k) : \{0,1\}^k \longrightarrow \{0,1\}$ outputs 1 if and only if $x_1 = x_2 = \ldots = x_k$. Consider the function $f : (\{0,1\}^n)^k \longrightarrow \{0,1\}$ given by $f(x_{ij}) = \oplus_{i=1}^n A(x_{i1}, \ldots, x_{ik})$.

   Prove that $C_k(f) \le O(k)$.