Complexity Theory B

Instructor: Alexander Razborov, University of Chicago
razborov@cs.uchicago.edu
Course Homepage: www.cs.uchicago.edu/~razborov/teaching/spring10.html

Spring Quarter, 2010

`You are encouraged to work together on solving homework problems, but please put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class.`

Homework 3, due June 3

1. Prove that a symmetric Boolean function $f(x_1, \ldots, x_n)$ can be represented as a threshold function $f(x) = \text{sign}(w_1 x_1 + \ldots + w_n x_n - t)$ $(w_1, \ldots, w_n, t \in \mathbb{R})$ if and only if it is a *Boolean* threshold function or its negation (that is, either $f(x) = 1$ iff $|\{ i \mid x_i = 1 \}| \geq t$ or $f(x) = 1$ iff $|\{ i \mid x_i = 1 \}| \leq t$ for some t).

2. Let $f : \{0,1\}^* \longrightarrow \{0,1\}^*$ be a one-way function and $g : \{0,1\}^* \longrightarrow \{0,1\}^*$ be a one-way permutation.

   (a) Prove that $f \circ g$ $(f \circ g(x) \stackrel{\text{def}}{=} f(g(x)))$ is a one-way function;

   (b) identify the reason why your argument fails if g is not necessarily a permutation.

3. A one way-function f is *perfect square* if for every binary string x, $f(x)$ is a perfect square (in binary representation).

   Prove that the existence of one-way functions implies the existence of perfect square one-way functions.

4. Prove that $L(f) \geq \Omega(\text{as}(f)^2)$, where L is the formula size in de Morgan basis and $\text{as}(f)$ is the average (over inputs) sensitivity of f. Explain why it implies that Khrapchenko's lower bound proof is natural.

5. Exercise 12.3 from Arora, Barak book, except for the last ($R(f) \geq \Omega(n)$) part.

   `Note.` Determining the *approximate* degree $\widetilde{\deg}(f)$ of that function is still a challenging open problem; see [27].