

Honors Discrete Mathematics

Instructor: Alexander Razborov, University of Chicago
razborov@math.uchicago.edu

Course Homepage: www.cs.uchicago.edu/~razborov/teaching/autumn17.html

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Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Shopping for solutions on the Internet is strongly discouraged: if you do it nonetheless, you *must* cite your source and, as the very least, explain the solution in your own words.

Homework is due at the beginning of class *unless* submitted by e-mail as a PDF file prepared from a TeX source. Electronic submissions conforming to these standards (no scans please!) are encouraged and accepted until Wednesday midnight at the Canvas.

Homework 4, due November 1

- (a) Prove that a function $f : X \rightarrow Y$ is injective if and only if for any finite set Z and any function $g : X \rightarrow Z$ there exists a function $h : Y \rightarrow Z$ such that $g = h \circ f$.
(b) Formulate and prove a similar criterion for surjective functions.
- Give a close form expression for the number of ordered pairs of n -digit integers that differ in every digit.
- Count the number of strings of length five consisting of Latin letters in which all letters are pairwise different, no two vowels are adjacent to each other and not two consonants are adjacent to each other.
- Prove that

$$\sum_{k=0}^n k \binom{n}{k}^2 = \frac{n}{2} \binom{2n}{n}.$$

5. (challenge problem)

(a) Prove that

$$\left| \binom{n}{0} - \binom{n}{2} + \binom{n}{4} - \binom{n}{6} \dots \right| \leq 2^{n/2}.$$

(b) Find an integer $n \geq 10^{100}$ for which the left-hand side is 0, and find another integer $n \geq 10^{100}$ for which this bound is tight.