Honors Discrete Mathematics

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Prove all of your answers with reasonable degree of mathematical rigor. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Shopping for solutions on the Internet is strongly discouraged: if you do it nonetheless, you *must* cite your source and, as the very least, explain the solution in your own words.

Homework is due at the beginning of Wednesday class *unless* submitted by e-mail as a PDF file prepared from a TeX source. Electronic submissions conforming to these standards (no scans please!) are encouraged and accepted until Wednesday midnight by Leo at lenacore@uchicago.edu.

Homework 4, due November 6

1. Give a closed form expression for the number of **unordered**[1] pairs (A, B), where $A, B \subseteq [n]$ are such that $|A \cap B| = 5$.

2. Prove that
$$\binom{n}{0} - \binom{n}{2} + \binom{n}{4} - \dots$$
is equal to $2^{n/2}$ if $8|n$, is equal to $-2^{n/2}$ if $n \equiv 4 \pmod 8$ and is equal to 0 if $n \equiv \pm 2 \pmod 8$.

3. Prove the inequality
$$S(m_1 + m_2, n) \geq \sum_{n_1=1}^{n-1} S(m_1, n_1) S(m_2, n - n_1)$$

[1] (A, B) and (B, A) are considered the same, A and B need not necessarily be different

(like with binomial coefficients, we set $S(m, n) \stackrel{\text{def}}{=} 0$ whenever $n > m$).

4. Let A_i ($i \in \mathbb{Z}_5$) be such that for any $i \in \mathbb{Z}_5$ we have $|A_i| = 100$, $A_i \cap A_{i+1} = \emptyset$ and $|A_i \cap A_{i+2}| = 10$. Compute $|A_0 \cup A_1 \cup \ldots \cup A_4|$.

5. Prove that $p_n(m)$ (the number of partitions of m using *at most* n numbers) is also equal to the number of partitions of $m + n$ using *exactly* n numbers.