

Honors Discrete Mathematics

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Course Homepage: www.cs.uchicago.edu/~razborov/teaching/autumn19.html

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Prove all of your answers with reasonable degree of mathematical rigor. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Shopping for solutions on the Internet is strongly discouraged: if you do it nonetheless, you *must* cite your source and, as the very least, explain the solution in your own words.

Homework is due at the beginning of Wednesday class *unless* submitted by e-mail as a PDF file prepared from a TeX source. Electronic submissions conforming to these standards (no scans please!) are encouraged and accepted until Wednesday midnight by Leo at lenacore@uchicago.edu.

Homework 5, due November 13

1. How many ways of distributing the standard 32 chess pieces into four distinguishable pouches are there? Pieces of the same kind and color (like two black rooks) are considered indistinguishable; the pouches are allowed to be empty.

Note: for the first two problems we want a plain integer as an answer. But once you come with a closed form expression, you may just evaluate it using a calculator without further explanations.

2. How many solutions in positive even integers are there to the equation

$$x_1x_2x_3 = 6^{10}?$$

Order matters.

3. Seventeen distinct points are chosen inside the 4-dimensional unit cube $[0, 1]^4$. Prove that there are two points x and y among them within the Euclidean distance $\left(\rho(x, y) \stackrel{\text{def}}{=} \sqrt{\sum_{i=1}^4 (x_i - y_i)^2}\right)$ at most one of each other.
4. We roll a (fair cubical) die 10 times. Prove that the probability to get the total at most 21 is equal to the probability to get the total at least 49.
5. Construct an example of five events E_i ($i \in \mathbb{Z}_5$) (in a sample space of your choice) such that for any $i \in \mathbb{Z}_5$, the events E_i and E_{i+1} are independent while E_i and E_{i+2} are not.