# Honors Discrete Mathematics

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Prove all of your answers with reasonable degree of mathematical
rigor (feel free to ask us when in doubt).  If you work with others
put their names clearly at the top of the assignment, everyone must
turn in their own independently written solutions.  Shopping for
solutions on the Internet is strongly discouraged.  But if you accidentally
stumble across it, then it will be accepted as long as you cite
the source and explain the solution in your own words.
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## Homework 3, due October 27

1. Compute $\gcd(2\sqrt{11}+1, 2021)$ in the ring $\mathbb{Z}[\sqrt{11}]$.
2. Consider the following binary relation $R(a,b)$ on $\mathbb{Q}$: $R(a,b)$ is true if and only if $a^7 - b^7 = a^3 - b^3$. Prove that $R$ is an equivalence relation and that every equivalence class is finite.
3. Prove that $\phi(m^2) = m \cdot \phi(m)$ ($\phi$ is Euler's function). Try to do it directly from the definition, by-passing prime factorization.
4. Which of the following four statements are always true? Prove those and give counterexamples to all others.
   (a) If the composition $f \circ g$ of two functions $f$ and $g$ is injective then $f$ is injective.
   (b) If $f \circ g$ is injective then $g$ is injective.
   (c) If $f \circ g$ is surjective then $f$ is surjective.
   (d) If $f \circ g$ is surjective then $g$ is surjective.

5. Solve the following system of congruences:

$$\begin{cases} x \equiv 1 \pmod{2} \\ x \equiv 0 \pmod{3} \\ x \equiv 1 \pmod{5} \\ x \equiv 0 \pmod{7} \\ x \equiv 1 \pmod{11} \\ x \equiv 0 \pmod{13}. \end{cases}$$