Honors Discrete Mathematics

Instructor: Alexander Razborov, University of Chicago
razborov@uchicago.edu
Course Homepage: www.cs.uchicago.edu/~razborov/teaching/autumn21.html

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Prove all of your answers with reasonable degree of mathematical
rigor (feel free to ask us when in doubt).  If you work with others
put their names clearly at the top of the assignment, everyone must
turn in their own independently written solutions.  Shopping for
solutions on the Internet is strongly discouraged.  But if you accidentally
stumble across it, then it will be accepted as long as you cite
the source and explain the solution in your own words.
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Homework 4, due November 3

1. Give a closed-form expression for the number of ordered triples $\langle A, B, C\rangle$ of subsets of $[n]$ such that $A \cap B \subseteq C$.

2. Prove the binomial identity
$$\sum_{k=2021}^{n} \binom{k}{2021}\binom{n}{k} = \binom{n}{2021} \cdot 2^{n-2021}.$$

3. Prove that
$$\left|\binom{n}{0} - \binom{n}{2} + \binom{n}{4} - \binom{n}{6} + \cdots\right| \leq 2^{n/2}.$$

4. Give a closed-form expression for the number of sets $A \in \binom{[n]\times[n]}{n+1}$ whose projection onto the first coordinate[1] is *full* (equal to $[n]$).

5. Prove that $S(2m, 2n) \geq S(m, n)^2$ for any positive integers m, n.

[1] that is, $\{x \in [n] \mid \exists y \in [n]((x, y) \in A)\}$