Honors Discrete Mathematics

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`Prove all of your answers with reasonable degree of mathematical rigor (feel free to ask us when in doubt). If you work with others put their names clearly at the top of the assignment, everyone must turn in their own independently written solutions. Shopping for solutions on the Internet is strongly discouraged. But if you accidentally stumble across it, then it will be accepted as long as you cite the source and explain the solution in your own words.`

Homework 5, due November 10

1. How many ways are there to choose six cards out of a standard deck of 52 cards in such a way that there are precisely three face cards[1], and all face cards are of pairwise different suits? Order of the cards in the selection does not matter. `In order to qualify for full credit, the answer must be in the form of a plain integer.`

2. The Latin alphabet consists of 5 vowels and 21 consonants. How many strings of length six are there in which all letters are pairwise different, no two vowels are adjacent to each other and not two consonants are adjacent to each other? `In order to qualify for full credit, the answer must be in the form of a plain integer.`

3. Give a closed-form expression for the number of solutions to the equation
$$x_1 \cdot \ldots \cdot x_n = 2^\ell 3^m,$$
where x_i are **even** positive integers. Order matters.

[1] Face card: J,Q or K

4. Prove that $p_n(m)$ (the number of partitions of m using *at most* n numbers) is equal to the number of partitions of $m+n$ using *exactly* n numbers.

5. Five points are chosen inside an equilateral triangle with side 1. Prove that there is a pair of them at the distance $\leq 1/2$ of each other.