Honors Discrete Mathematics

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Prove all of your answers with reasonable degree of mathematical rigor (feel free to ask us when in doubt). If you work with others put their names clearly at the top of the assignment, everyone must turn in their own independently written solutions. Shopping for solutions on the Internet is strongly discouraged. But if you accidentally stumble across it, then it will be accepted as long as you cite the source and explain the solution in your own words.

Homework 6, due November 17

1. A fair (cubical) die is rolled n times. Give a closed-form expression for the probability that

   (a) the sum

   (b) the product

   of outcomes is equal to 4 mod 6.

2. Construct an example of 2021 events in the same sample space that are pairwise independent but such that no three of them are mutually independent.

3. A person plays a slot machine with probability p of a win until either two wins occur in a row or two losses occur in a row (and then goes home). Compute, as a closed-form expression, the distribution of the random variable that is equal to the number of plays in this game.

4. Let X and Y be two independent random variables on the same sample space with values in $\mathbb{Z}_n^*$ (n an integer) and assume that X is *uniformly* distributed. In other words, $p(X = a) = \frac{1}{\phi(n)}$ for any $a \in \mathbb{Z}_n^*$, where $\phi(n) = |\mathbb{Z}_n^*|$ is the Euler function. Compute, as a closed-form expression, the distribution of the product XY.

5. Compute, as a closed-form expression, the expectation $E(X/Y)$, where X and Y are independent random variables uniformly distributed on the set $\{1, 2, \ldots, 2^n\}$.