Honors Discrete Mathematics

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`Prove all of your answers with reasonable degree of mathematical rigor (feel free to ask us when in doubt). If you work with others put their names clearly at the top of the assignment, everyone must turn in their own independently written solutions. Shopping for solutions on the Internet is strongly discouraged, and using AI or StackExchange` **shall be considered academic dishonesty and treated appropriately.**

Homework 1, due October 16

1. Let $a_0 = -1$ and $a_{n+1} = \sqrt{\frac{a_n+1}{2}}$ $(n \geq 0)$. Give a closed form expression for a_n.

   `Hint. You may want to refresh your knowledge of trigonometry.`

2. Consider the rectangular grid $[n] \times [m]$ as a set of points on the plane. What is the minimum number of lines needed to cover all these points? Prove *both* the lower and upper bound of course.

3. Prove or disprove the following. Let $f_1, g_1, f_2, g_2 : \mathbb{N} \longrightarrow \mathbb{N}$ be functions such that $\forall n(f_1(n) > f_2(n))$, $\forall n(g_1(n) > g_2(n))$, $f_1(n) \leq O(g_1(n))$ and $g_2(n) \leq O(f_2(n))$. Then $f_1(n) - f_2(n) \leq O(g_1(n) - g_2(n))$.

4. Consider the two linear orderings $(\mathbb{R}_-, \leq)$ and $(\mathbb{R}_+, \leq)$, i.e. restrictions of the standard ordering on the reals to negative and positive numbers, respectively. Prove that they are isomorphic.

5. Prove that an ordinal α is infinite if and only if $1 + \alpha \neq \alpha + 1$.