Honors Discrete Mathematics

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Prove all of your answers with reasonable degree of mathematical rigor (feel free to ask us when in doubt). If you work with others put their names clearly at the top of the assignment, everyone must turn in their own independently written solutions. Shopping for solutions on the Internet is strongly discouraged, and using AI or StackExchange **shall be considered academic dishonesty and treated appropriately.**

Homework 4, due November 6

1. How many ways are there to choose six cards out of a standard deck of 52 cards in such a way that all face cards (J,Q,K) among the chosen are red (Diamonds or Hearts)? Order does not matter.

   Note. To qualify for full credit, the final answer in this problem must be a plain integer.

2. Prove the binomial identity

$$\sum_{i=\ell}^{n} \binom{i}{\ell}\binom{n}{i} = \binom{n}{\ell} \cdot 2^{n-\ell} \ (n \geq \ell).$$

3. (a) Prove that for all m_1, m_2, n we have

$$S(m_1+m_2, n) \geq \left(\sum_{j=1}^{n} P(n,j)S(m_1,j)\right) \cdot S(m_2,n);$$

   (b) explain why this inequality is not tight.

4. Prove that for any $n \geq 1$,

$$\sum_{\substack{k,\ell \geq 0 \\ k+\ell \leq n}} 2^{k+\ell} \binom{n}{k\ \ell\ n-k-\ell} \Bigg| \sum_{\substack{k,\ell \geq 0 \\ k+\ell \leq n}} 7^{k+\ell} \binom{n}{k\ \ell\ n-k-\ell}.$$

5. Let $m > n$. Prove that the number of partitions of m in which the largest number is n is equal to $p_n(m-n)$ (the number of partitions of $m-n$ using at most n numbers).