Honors Discrete Mathematics

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`Prove all of your answers with reasonable degree of mathematical rigor (feel free to ask us when in doubt). If you work with others put their names clearly at the top of the assignment, everyone must turn in their own independently written solutions. Shopping for solutions on the Internet is strongly discouraged, and using AI or StackExchange is strictly prohibited, it` **shall be considered academic dishonesty and treated accordingly.**

Homework 4, due November 5

1. How many ways are there to choose 14 cards out of a standard deck of 52 cards in such a way that all 13 values (2-A) are represented in the selection?

2. Compute
$$\binom{2025}{0} - \binom{2025}{2} + \binom{2025}{4} - \dots$$

3. Let $A_1, \dots, A_{2n}$ be sets such that $|A_i| = m$ $(1 \le i \le 2n)$ and $|A_i \cap A_j| = (i - j) \mod 2$ for all $1 \le i < j \le 2n$. Compute $|A_1 \cup A_2 \dots \cup A_{2n}|$.

4. How many equivalence relations on $[3n]$ in which every class contains precisely 3 elements are there?

5. How many solutions does the equation
$$x_1 + x_2 + \dots + x_{3n} = m$$
have, where m is a fixed non-negative integer and x_i run over non-negative integers such that $x_i \equiv i \mod 3$. Order matters.