Quantum Computing

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Homework 1, due May 7

1. Prove that the TOFFOLI gate can not be realized as a *reversible* circuit with gates acting on at most two bits.

   `Hint. Reversible gates on two bits have one very special property; identify it.`

2. For which values $N \leq 11$ does there exist an $N \times N$ unitary matrix in which all entries are $\pm\frac{1}{\sqrt{N}}$?

3. We have seen in class how to implement the oracle $V_f:\ V_f\left|x\right\rangle = (-1)^{f(x)}\left|x\right\rangle$ using the oracle $U_f:\ U_f\left|x,y\right\rangle = \left|x, y\oplus f(x)\right\rangle$.

   (a) Prove that the opposite is impossible.

   (b) Implement U_f using the controlled version of V_f, i.e., the operator given by

   $$\Lambda(V_f) : \left|x,y\right\rangle \longrightarrow \begin{cases} \left|x,y\right\rangle & \text{if } y=0 \\ (-1)^{f(x)}\left|x,y\right\rangle & \text{if } y=1. \end{cases}$$

4. Prove that an N-dimensional operator A is normal if and only if $A^\dagger$ can be represented as a linear combination of $I_N, A, A^2, \ldots, A^{N-1}$.

   `Hint. The solution I have in mind uses Lagrange's interpolation. But you should feel free to shop for a solution in textbooks on linear algebra as well.`

5. Prove that $QFT_{mn} \approx QFT_m \otimes QFT_n$ if and only if m and n are relatively prime. Here $\approx$ stands for the unitary equivalence: $A \approx B$ if and only if there exists a unitary operator P such that $B = PAP^\dagger$.