Quantum Computing

Instructor: Alexander Razborov, University of Chicago.
razborov@uchicago.edu
Course Homepage: www.cs.uchicago.edu/~ razborov/teaching/winter22.html

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You may work together on solving homework problems, but please put all the names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Shopping for solutions on the Internet is strongly discouraged but if you do it anyway, you must completely understand the proof, explain it in your own words and include the URL. On the contrary, shopping for useful facts is encouraged.

PDF file **prepared from a TeX source** is very much preferred format. In that case you will get back your feedback in equally neat form.

Homework 1, due February 11

1. Show how to realize the Fredkin gate[1] by a reversible circuit consisting of three Toffoli gates.

2. Prove that a real (square) matrix is both stochastic and orthogonal if and only if it is a permutation matrix.

3. Let U and V be linear operators in finite-dimensional Hilbert spaces. Prove that $U \otimes V$ is unitary if and only if there exists $\alpha \neq 0$ such that both αU and $\alpha^{-1} V$ are unitary.

4. What will happen if we try to run vanilla[2] Grover's search on a function f for which $|f^{-1}(0)| = 1$, i.e. all inputs but one are good solutions? Say, will the success probability still be $\geq 1/2$ or not?

[1] Recall that $(0, y, z) \stackrel{F}{\mapsto} (0, y, z)$, $(1, y, z) \stackrel{F}{\mapsto} (1, z, y)$.
[2] our basic version for finding *unique* solutions

5. Let N be a prime, $a \in \mathbb{Z}_N^*$ and $b \in \mathbb{Z}_N$. Prove that the operators $U_a : |x\rangle \mapsto |ax\rangle$ and $V_b : |x\rangle \mapsto |x + b\rangle$ $(x \in \mathbb{Z}_N)$ have exactly one common eigenvalue and, moreover, they also share an eigenvector corresponding to this eigenvalue.