Quantum Computing

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You may work together on solving homework problems, but put all the names of your collaborators clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Shopping for solutions on the Internet is strongly discouraged but if you do it anyway, you must completely understand the proof, explain it in your own words and include the URL. On the contrary, shopping for useful facts is encouraged.

Prove all your answers of course.

PDF file **prepared from a TeX source** is very much preferred format. In that case you will get back your feedback in a neat annotated form.

Homework 1, due February 3

1. Recall that the Fredkin gate is given by

$$(x, y, z) \overset{F}{\mapsto} \begin{cases} (x, y, z) & \text{if } x = 0 \\ (x, z, y) & \text{if } x = 1. \end{cases}$$

   Define also the TWISTED SWAP[1] gate by

$$(x, y) \overset{TS}{\mapsto} \begin{cases} (x \oplus 1, y \oplus 1) & \text{if } x = y \\ (x, y) & \text{if } x \neq y. \end{cases}$$

   Prove that no reversible circuit with ancilla bits using these two gates can realize the NOT gate if we insist on garbage removal. In other words, prove that there does not exist $L \geq 1$ and a reversible circuit in the basis $\{F, TS\}$ that swaps the strings 0^{L+1} and 10^L.

[1] this name is my invention

2. Can the sum of two unitary operators ever be unitary?

3. For any two Boolean functions $f, g : \{0,1\}^n \longrightarrow \{0,1\}$ we clearly have $U_f^* U_g^* = U_{f \oplus g}^*$. Assume now that you have an unlimited supply of black boxes U_f^*, U_g^*, an unlimited number of ancilla qubits and you need not clean up the garbage. Can you realize $U_{f \wedge g}^*$ in the same manner?

4. If we feed into Deutch-Josza algorithm the parity function $f(x_1, \ldots, x_n) = x_1 \oplus \ldots \oplus x_n$ then (since it is balanced) the probability that the measurement will produce 0^n is zero. Completely describe the final state that will be actually produced by the algorithm.

5. Let $A : \mathbb{C}^N \longrightarrow \mathbb{C}^N$ be a normal operator. Which of the following two statements are true:

   (a) if A^3 is unitary then A is also unitary;

   (b) if A^3 is Hermitian then A is also Hermitian.