Quantum Computing

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Homework 2, due May 23

1. Prove that for any two quantum states $|\phi\rangle$ and $|\psi\rangle$ in the same dimension,
$$|\langle\phi|\psi\rangle| \geq 1 - \frac{||\,|\phi\rangle - |\psi\rangle\,||^2}{2}.$$

2. Let f be the Boolean function in $2n^2$ variables x_{ij}, y_{ij} $(1 \leq i, j \leq n)$ defined as follows: $f(\vec{x}, \vec{y}) = 1$ iff there exist $i, j \in \{1, \ldots, n\}$ such that $x_{ij} = y_{ij} = 1$ while $x_{ij'} = y_{ij'} = 0$ for any other $j' \neq j$.

   Compute the sensitivity $\mathbf{s}(f)$ and the block sensitivity $\mathbf{bs}(f)$ of this function.

3. $\mathrm{CLIQUE}_{3,N}$ is the Boolean function in $\binom{N}{2}$ variables encoding an N-vertex graph that outputs one if and only if this graph contains a triangle. Prove that
$$\Omega(N) \leq Q_2(\mathrm{CLIQUE}_{3,N}) \leq O(N^{3/2}).$$

4. Let
$$f(x,y) = \begin{cases} 1 & \text{iff } x \leq y \leq x + 2013 \\ 0 & \text{otherwise} \end{cases}$$
   (we view x and y as integers in $\{0, 1, \ldots, 2^n - 1\}$). Prove that the determinstic communication complexity of this predicate is $\Omega(n)$.