Quantum Computing

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Winter Quarter, 2018

Homework 2, due February 22 at the beginning of the class. LaTeX submissions are strongly encouraged and accepted until Friday 12pm.

1. Let $\mathcal{H}$ be a Hilbert space with $\dim(\mathcal{H}) \geq 2$. Does there exist a Hermitian bi-linear[1] mapping $C : \mathcal{H} \times \mathcal{H} \longrightarrow \mathcal{H}$ such that for any two states (unit vectors) $u, v \in \mathcal{H}$, $C(u, v)$ is again a unit vector?

2. Let us call a Boolean string $a_1, \ldots, a_N$ *mid-Western* if there exists at least one index C such that $a_C = a_{C+1} = 1$ while $a_M = 0$ for any other M with $|C - M| \leq \sqrt{N}$. Let $MW_N(X_1, \ldots, X_N)$ be the characteristic function of the set of all mid-Western strings.

   Determine its sensitivity and block-sensitivity within a multiplicative constant.

3. Prove that $\widetilde{\deg}_{0.01}(F) \leq O(\widetilde{\deg}(F))$ for any total function F, where $\widetilde{\deg}_\epsilon$ is defined just as $\widetilde{\deg}$, replacing $1/3$ with ϵ.

4. Let $Q_0(F)$ and $Q_1(F)$ be the quantum query complexity with no error (the algorithm must output the correct value with probability 1) and with one-sided error (no error on 1-inputs and error $\leq 1/3$ on 0-inputs), respectively.

   Find a total function $F(X_1, \ldots, X_N)$ that exhibits a super-linear gap between $Q_0(F)$ and $Q_1(F)$.

5. $\mathrm{CLIQUE}_{k,N}$ is the Boolean function in $\binom{N}{2}$ variables encoding an N-vertex graph that outputs one if and only if this graph contains a

[1] see Problem 3 in the previous homework

complete subgraph on k vertices. Prove that

$$\Omega(N) \leq Q_2(\text{CLIQUE}_{3,N}) \leq O(N^{3/2}).$$