Quantum Computing

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`You may work together on solving homework problems, but please put all the names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Shopping for solutions on the Internet is strongly discouraged but if you do it anyway, you must completely understand the proof, explain it in your own words and include the URL. On the contrary, shopping for useful facts is encouraged.`

`PDF file` **prepared from a TeX source** `is very much preferred format. In that case you will get back your feedback in equally neat form.`

Homework 2, due February 25

1. Let $\approx$ stand for the unitary equivalence: $A \approx B$ if and only if there exists a unitary operator U such that $B = UAU^\dagger$. Find examples of $m, n \geq 2$ such that $QFT_{mn} \not\approx QFT_m \otimes QFT_n$ and:

   (a) m and n are relatively prime,

   (b) m and n are not relatively prime.

2. Let us call a Boolean string $A_1, \ldots, A_N$ *mid-Western* if there exists at least one index c such that $A_c = A_{c+1} = 1$ while $A_m = 0$ for any other m with $|c - m| \leq \sqrt{N}$. Let $MW_N(X_1, \ldots, X_N)$ be the characteristic function of the set of all mid-Western strings.

   Determine its sensitivity and block-sensitivity within a multiplicative constant.

3. Assume that for some d, s there exists a total Boolean function $F(X_1, \ldots, X_N)$ with $\deg(F) = d$ and $\mathsf{s}(F) \leq s$. Prove that there exists another function with these properties that depends on exactly d variables.

4. (a) Compute $\widetilde{\deg}(X_1 \wedge X_2)$.

   (b) Let $\epsilon \stackrel{\text{def}}{=} 0.01$ and define $\widetilde{\deg}_\epsilon$ similarly to $\widetilde{\deg}$, replacing $1/3$ with ϵ. Describe all total Boolean functions F with $\widetilde{\deg}_\epsilon(F) = 1$.

5. $\text{CLIQUE}_{3,N}$ is the Boolean function in $\binom{N}{2}$ variables encoding an N-vertex graph that outputs one if and only if this graph contains a triangle. Prove that

$$\Omega(N) \leq Q_2(\text{CLIQUE}_{3,N}) \leq O(N^{3/2}).$$

`Note. Narrowing this gap is a long-standing open problem for which known methods seem to fail completely.`