Quantum Computing

Instructor: Alexander Razborov, University of Chicago.
razborov@uchicago.edu
Course Homepage: www.cs.uchicago.edu/~ razborov/teaching/winter23.html

Winter Quarter, 2023

You may work together on solving homework problems, but put all the names of your collaborators clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Shopping for solutions on the Internet is strongly discouraged but if you do it anyway, you must completely understand the proof, explain it in your own words and include the URL. On the contrary, shopping for useful facts is encouraged.

Prove all your answers of course.

PDF file **prepared from a TeX source** is very much preferred format. In that case you will get back your feedback in a neat annotated form.

Homework 2, due February 17

1. Let $U \in \mathbb{U}(n)$ be an $n \times n$ unitary matrix. Complete the following sentence using four mathematical symbols.

   There exist $m \geq 1$, $W \in \mathbb{U}(m)$ and $V \in \mathbb{U}(n+m)$ such that

   $$\begin{pmatrix} U & 0 \\ 0 & W \end{pmatrix} = V^\dagger QFT_{m+n} V$$

   if and only if ...

2. (a) Prove that $C^{(1)}(F) + C^{(0)}(F)$ (where $C^{(0)}(F)$ is the 0-certificate complexity) is polynomially equivalent to all complexity measures in our mega-theorem.

   (b) Will it remain true for $C^{(1)}(F)$ itself?

3. Let us call a Boolean string $A_1, \ldots, A_N$ *mid-Western* if there exists at least one index c such that $A_c = A_{c+1} = 1$ while $A_m = 0$ for any other m with $|c - m| \leq \sqrt{N}$. Let $MW_N(X_1, \ldots, X_N)$ be the characteristic function of the set of all mid-Western strings.

   Determine its sensitivity and block-sensitivity within a multiplicative constant.

4. $\mathrm{CLIQUE}_{3,N}$ is the Boolean function in $\binom{N}{2}$ variables encoding an N-vertex graph that outputs one if and only if this graph contains a triangle. Prove that

$$\Omega(N) \leq Q_2(\mathrm{CLIQUE}_{3,N}) \leq O(N^{3/2}).$$

   `Note. Narrowing this gap is a long-standing open problem for which known methods seem to fail completely.`

5. Prove that $Q_0(X_1 \oplus \ldots \oplus X_N) = Q_2(X_1 \oplus \ldots \oplus X_N)$, where $Q_0(F)$ is the zero-error quantum query complexity.