Quantum Computing

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Homework 3, due March 7

1. Prove or disprove that

$$QC_2(f \circ \wedge^n) \le \widetilde{O}(QC_2(f \circ \oplus^n)),$$

   where $f : \{0,1\}^n \longrightarrow \{0,1\}$ is a total function.

2. Let X be an $(N \times N)$ matrix such that $\mathrm{Tr}(X\rho) = 1$ for any density matrix ρ. Prove that $X = I_N$.

3. Alice and Bob play the following (complete information) game. Alice picks a mixed state ρ in a 1-qubit space, after that Bob picks another mixed state σ, and then Alice pays Bob $D(\rho, \sigma)$ dollars. Determine the value of this game, the best strategy for Alice and describe the set of best responses for Bob.

4. Consider the projective measurement[1] T corresponding to the orthogonal decomposition

$$\mathcal{H} = \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \ldots \oplus \mathcal{H}_L.$$

   Describe **explicitly** a decomposition of T as an isometric embedding followed by tracing-out.

[1] as we did in class, the classical outcome is discarded