Quantum Computing

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Course Homepage: www.cs.uchicago.edu/~ razborov/teaching/winter22.html

Winter Quarter, 2022

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You may work together on solving homework problems, but please
put all the names clearly at the top of the assignment.  Everyone
must turn in their own independently written solutions.  Shopping
for solutions on the Internet is strongly discouraged but if you
do it anyway, you must completely understand the proof, explain
it in your own words and include the URL. On the contrary, shopping
for useful facts is encouraged.
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PDF file **prepared from a TeX source** is very much preferred format. In that case you will get back your feedback in equally neat form.

Homework 3, due March 11

1. Let

$$f(x,y) = \begin{cases} 1 & \text{iff } x \leq y \leq x + 2022 \\ 0 & \text{otherwise} \end{cases}$$

   (we view x and y as integers in $\{0, 1, \ldots, 2^n - 1\}$). Prove that the probabilistic communication complexity of this predicate is $O(\log n)$.

2. Let us switch the roles of $\oplus$ and $\wedge$ in IP_n and consider the following function:

$$f_n(x,y) = \bigwedge_{i=1}^{n} (x_i \oplus y_i).$$

   Determine $QC_2(f_n)$ up to a logarithmic factor.

   For extra credit: determine this quantity up to a constant factor.

3. Prove that for any normal $N \times N$ matrix M,

$$||M||_{\text{tr}} = \max \{ |\langle M, U \rangle| \mid U \in \mathbb{U}(N) \} ,$$

where $\mathbb{U}(N)$ is the group of unitary operators.

For extra credit: prove this for arbitrary square complex matrices.

4. For a density matrix ρ, define its *pureness* as the maximum p for which ρ can be represented as a mixed state of the form $\{(|\phi_1\rangle, p_1), \ldots, (|\phi_t\rangle, p_t)\}$ with $p_1 = p$.

Describe the pureness in terms of a matrix norm of ρ.

5. Consider the one-qubit probabilistic quantum circuit that with equal probability $\eta < 1/3$ applies one of the Pauli matrices X, Y, Z (and does not do anything with probability $1 - 3\eta$). Give a (very!) explicit description of what this circuit computes.