

Quantum Computing

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Course Homepage: www.cs.uchicago.edu/~razborov/teaching/winter23.html

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You may work together on solving homework problems, but put all the names of your collaborators clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Shopping for solutions on the Internet is strongly discouraged but if you do it anyway, you must completely understand the proof, explain it in your own words and include the URL. On the contrary, shopping for useful facts is encouraged.

Prove all your answers of course.

PDF file **prepared from a TeX source** is very much preferred format. In that case you will get back your feedback in a neat annotated form.

Homework 3, due March 3

1. Alice and Bob hold two subsets $U, V \subseteq [n]$, and they want to determine whether $|U \cap V| \geq n/2$. Prove that the randomized communication complexity of this problem (with two-sided error) is $\Omega(n)$.
2. Find a symmetric Boolean function $f(z_1, \dots, z_n)$ **other than the parity function** for which $QC_2(f \circ \oplus^n)$ is exponentially smaller than $QC_2(f \circ \wedge^n)$. Here we denote

$$(f \circ g^n)(x_1, \dots, x_n, y_1, \dots, y_n) \stackrel{\text{def}}{=} f(g(x_1, y_1), \dots, g(x_n, y_n)),$$

where $g : \{0, 1\}^2 \longrightarrow \{0, 1\}$.

3. Recall that a two-qubit state is *non-entangled* if it has the form $|\phi\rangle \otimes |\psi\rangle$, where $|\phi\rangle, |\psi\rangle$ are 1-qubit states.

Prove that there does not exist a probability distribution on non-entangled states that has the same density matrix as the EPR state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

4. Let $\mathcal{H} = \mathbb{C}^{2^n}$ be the space of states of an n -qubit system, $\mathcal{H} = \mathcal{H}_0 \dot{+} \mathcal{H}_1$ be its orthogonal decomposition corresponding to the first qubit. Decompose the corresponding projective measurement¹ **explicitly** as an isometric embedding followed by tracing-out.
5. A one-qubit random quantum circuit consists of a single unitary gate picked uniformly at random from the set of four Pauli matrices $\{I, X, Y, Z\}$. Describe the result of its application to a state $|\phi\rangle$ using at most four symbols (subscripts do not count again).

¹the classical outcome is discarded