

Graph Theory

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Course Homepage: www.cs.uchicago.edu/~razborov/teaching/spring12.html

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You are encouraged to work together on solving homework problems, but please put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class.

Homework 2, due April 18

1. Let G be a connected graph with 1984 vertices and 2012 edges. Prove that G contains at least 29 different cycles¹ and give an example showing that this bound is tight.
2. Let C_n be a cycle on n vertices. How should we append one more edge e to C_n to maximize the number $\tau(C_n + e)$ of the spanning trees in the resulting graph? Compute this maximal value.
3. Is the number of spanning trees in the graph on Figure 1 even or odd?
4. Prove that the number of different (that is, up to an isomorphism) trees on n vertices is at least $\frac{n^{n-2}}{n!}$.
5. Construct a simple graph G with $\kappa(G) = 3$, $\kappa'(G) = 4$ and $\delta(G) = 5$.

¹remember that for the purposes of this class “cycle” always means a cycle in which vertices do not repeat themselves

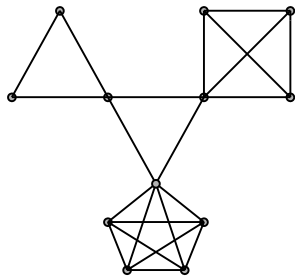


Figure 1: Just a nice graph