Graph Theory

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You are encouraged to work together on solving homework problems,
but please put their names clearly at the top of the assignment.
Everyone must turn in their own independently written solutions.
Homework is due at the beginning of class.
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Homework 3, due April 25

1. Let G and H be two simple graphs on the same set of vertices V. We say that G is *better connected* than H if for every $V_0 \subseteq V$ the following is true: if removing V_0 from G disconnects G, then also removing V_0 from H disconnects H.

   Prove that G is better connected than H **if and only if** $E(H) \subseteq E(G)$.

2. Prove that there exists an orientation of the 4-cube graph Q_4 such that all out-degrees $d^+(v)$ are odd.

3. Prove that if a graph on n vertices is isomorphic to its complement, then n is of the form $4k$ or $4k + 1$.

4. Find the smallest tree (and prove that it is indeed the smallest) with at least two vertices that has only one automorphism (namely, the identity).

5. Recall that a graph G is *vertex-transitive* if for every $u, v \in V(G)$ there is an automorphism f of G such that $f(u) = v$. Prove that every connected vertex-transitive graph is either a single vertex or a single edge or is 2-connected.