

Graph Theory

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Course Homepage: www.cs.uchicago.edu/~razborov/teaching/spring12.html

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You are encouraged to work together on solving homework problems, but please put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class. To earn full credit, you must prove all of your answers.

Homework 6, due May 16

1. Prove that $\alpha(T) \geq n/2$ for any tree T .
2. A *tournament* is an orientation of a complete graph. A tournament T is *transitive* if it is acyclic (equivalently, $V(T)$ can be ordered in such a way $\{v_1, \dots, v_n\}$ that all arcs are directed from v_i to v_j with $i < j$). Prove that every tournament with 2^{k-1} vertices contains a transitive subtournament on k vertices.
3. Compute $\text{ex}(n; K_4^3)$. In words: how many edges can a 3-graph on 5 vertices have if it is known that it does not contain any complete subgraph on 4 vertices.
4. Consider the graph G_n with the vertex set $\{0, 1, 2\}^n$ in which two vertices are connected if and only if they differ in precisely one coordinate. Prove that $\chi(G_n) = 3$.
5. Let G be a graph on 5 vertices. Prove that if $\omega(G) = \alpha(G)$ then $\chi(G) = 3$.