Graph Theory

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You are encouraged to work together on solving homework problems, but please put your names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class. To earn full credit, you must prove all of your answers.

Homework 7, due May 23

1. Calculate[1] $\chi(C_5 \times_\ell C_5)$.
2. Compute the chromatic polynomial of the graph on Figure 1

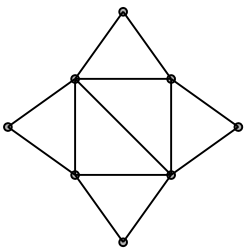

Figure 1: Just a graph.

3. For a finite set of points V on the plane, let G_V be the graph on V in which u and v are adjacent if and only if they are at (Euclidean) distance 1 from each other.

   Give an example of a set V with $\chi(G_V) = 4$.

[1] recall that $\times_\ell$ stands for the lexicographic product of graphs

4. Prove that the graph on Figure 2 is not planar.

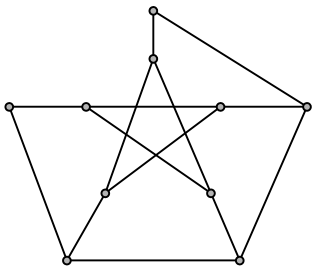

Figure 2: Petersen graph with a missing edge

5. Let G be a graph on n vertices such that G has K_k as a minor, but G is minimal with this property, i.e. no proper subgraph of G has K_k as a minor. Prove that G has $n + \frac{k(k-3)}{2}$ edges.