

Graph Theory

Instructor: Alexander Razborov, University of Chicago
razborov@cs.uchicago.edu

Course Homepage: www.cs.uchicago.edu/~razborov/teaching/spring12.html

Spring Quarter, 2012

You are encouraged to work together on solving homework problems, but please put your names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class. To earn full credit, you must prove all of your answers.

Homework 8, due May 30

1. Describe all connected graphs on $n \geq 4$ vertices that do not¹ contain:
 - (a) P_3 as an *induced* subgraph;
 - (b) P_4 as a (not necessarily induced) subgraph.
2. Let G be the complement to a tree on 2012 vertices. Describe all possible values $\chi(G)$ may take.
3. What is the minimal possible total number of acyclic orientations a connected graph on n vertices may have? And what is the maximal number?
4. Construct two simple graphs G and H such that H is a minor of G , but not its subdivision.
5. Prove that K_ℓ is a minor of $K_{k,k}$ if and only if $\ell \leq k + 1$.

¹ P_ℓ is the path on ℓ vertices of length $\ell - 1$