Computability and Complexity Theory

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You are encouraged to work together on solving homework problems,
but please put their names clearly at the top of the assignment.
Everyone must turn in their own independently written solutions.
The due date below pertains to e-mail submissions as a PDF file,
which is the preferred format.  Handwritten work must be submitted
by the beginning of class a day before the deadline.
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Homework 1, due May 7

1. Let $f(x)$ be a *non-decreasing* total recursive function. Prove that $\mathrm{im}(f)$ is recursive.

2. Design a Markov algorithm for computing the function $f(x) = x \bmod 3$.

3. An URM M *b-accepts*[1] an input x if in the overall course of its computation on x, the register R_1 contains only finitely many different values (thus, if M halts on x, it always b-accepts, but the converse is not necessarily true). Let

$$L_{\mathrm{b}} \stackrel{\mathrm{def}}{=} \{(x, y) \mid M_x \text{ b-accepts } y\}$$

(M_x is the URM with code x).

   (a) Prove that every r.e. set is many-one reducible to L_{b}.

   (b) Prove that every r.e. set is many-one reducible to its complement $co - L_{\mathrm{b}}$.

[1] "b" stands for bounded

(c) Prove that L_{b} is not r.e.

4. Do there exist two r.e. sets A, B such that $A \subseteq B$ and:

   (a) A is creative while B is simple;

   (b) A is simple while B is creative?

5. Describe all total recursive functions $g(x)$ for which the operator $\Phi_g : \mathcal{F}_2 \longrightarrow \mathcal{F}_1$ defined by the formula $\Phi_g(f)(x) \stackrel{\text{def}}{=} g(\mu y(f(x, y) = 0))$ is effective.