Discrete Mathematics

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Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class (paper submission) or 11:59pm (PDF generated from a (La)TeX source, e-mailed to Leonardo).

Homework 7, due March 2

1. Prove that a connected graph with n vertices has n edges if and only if it contains *precisely* one simple cycle.

2. Prove that the number of labelled spanning trees in the complete bipartite graph $K_{3,n}$ is equal to $n^2 3^{n-1}$.

3. Consider the standard representation of rooted binary trees using two pointers (LEFT, RIGHT) at each vertex, containing the addresses of the left and right child, respectively. In class, we showed how to visit all vertices of the tree (say, in inorder) starting at the root, by using a stack of pointers of size equal to the depth of the tree.

   Sketch an algorithm that visits all vertices of a rooted binary tree with n vertices in inorder, uses only a constant number of pointers (and no stacks) other than the ones in the tree structure itself, and uses a constant number of integer variables whose maximum absolute value is cn for some constant c.

4. A graph G is *k-edge colorable* if there exists a function $f : E(G) \longrightarrow [k]$ such that $f(e) \neq f(e')$ whenever e and e' share a vertex. A *Hamiltonian cycle* in an n-vertex graph is a sub-graph isomorphic to C_n. Finally, a graph G is *d-regular* if $\deg_G(v) = d$ for every $v \in V(G)$.

Prove that every 3-regular graph that contains a Hamiltonian cycle is always 3-edge colorable.