Propositional Proof Complexity

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Homework 1

1. Prove that any contradiction τ has a resolution refutation. What is the obvious upper bound on its size?

2. Let G be the graph with $V(G) = \{1, 2, \dots, n\}$ and $E(G) = \{\,(i, j) \mid |i - j| \leq 17\,\}$. Prove that $\tau(G, f)$ has polynomial size resolution refutation for any odd function f. Try to generalize this result as wide as possible.

3. Deduce the uniqueness condition

   $$(((y+z = x) \vee (x \leq y \wedge z = 0)) \wedge ((y+z' = x) \vee (x \leq y \wedge z' = 0))) \Longrightarrow z = z'$$

   for the binary substraction function from the BASIC axioms (to be found on pages 30-31 of Buss's monograph).

4. T_2^k proves $\Pi_k^b - IND$. S_2^k proves $\Pi_k^b - PIND$.

5. Building on the known fact that $S_2^k(\alpha) \neq T_2^k(\alpha)$ (for any k), prove that $S_2^k(f) \neq T_2^k(f)$ (as usual, here α and f are new predicate and function symbols, respectively).