1. Let \( p, q, \) and \( r \) be propositions. Prove or disprove:

\[
p \land (q \lor r) \equiv (p \lor q) \land (p \lor r)
\]

2. Which of the following statements are true? For each false statement, provide a counterexample. The domain (universe) for all variables is the set of real numbers.

(1) \( \forall x \exists y \ (x^2 = y) \)
(2) \( \forall x \exists y \ (x = y^2) \)
(3) \( \exists x \forall y \ (xy = 0) \)
(4) \( \exists y \forall x \ (x + y = 0) \)
(5) \( \forall x \exists y \ (x + y = 1) \)

3. Let \( a, b, \) and \( c \) be integers. Prove that if \( a|b \) and \( a|c \), then \( a|(b + c) \).

4. Prove by mathematical induction the following summation formulas:

(1) \( 2 + 4 + \cdots + 2n = n(n + 1) \)
(2) \( 2^2 + 4^2 + \cdots + (2n)^2 = \frac{2n(n + 1)(2n + 1)}{3} \)

5. Evaluate each of the following sums:

(1) \( 5 + 25 + 125 + 625 + \cdots + 5^{10} \)
(2) \( \sum_{i=1}^{100} 2i \)
(3) \( \sum_{i=2}^{8} 3 \)
(4) \( \sum_{i=0}^{8} (2^{i+1} - 2^i) \)

6. Let \( A = \{a, b, c\} \) and \( B = \{x, y\} \), and \( C = \{0, 1\} \). Compute each of the following sets.

(1) \( A \times B \times C \)
(2) \( C \times B \times A \)
7. Let \( A, B, \) and \( C \) be sets. Prove or disprove:

\[
A - (B \cap C) = (A - B) \cap (A - C)
\]

8. (1) Can you conclude that \( A = B \) if \( A, B, \) and \( C \) are sets such that 
\( A \cup C = B \cup C \)? Justify your answer.

(2) Can you conclude that \( A = B \) if \( A, B, \) and \( C \) are sets such that 
\( A \cap C = B \cap C \)? Justify your answer.

9. Write down all of the properties that each of the following relations \( R \) 
defined on the set of positive integers satisfies from among the properties 
reflexive, symmetric, antisymmetric, and transitive. Justify your answers.

(1) \((x, y) \in R \) if 2 divides \( x + y \)

(2) \((x, y) \in R \) if 3 divides \( x + y \)

10. Give an example of a relation on \( \{1, 2, 3, 4\} \) that is reflexive, not antisymmetric, and not transitive.