Math. 314: Homework 7

1. Problem 14 from Rudin, Chapter 14: Let \( \Omega : \{ z = x + iy \mid y \in (-1, 1) \} \), \( f \in H(\Omega) \), \( |f| \leq 1 \) in \( \Omega \), and \( \lim_{x \to \infty} f(x) = 0 \). Prove that \( \lim_{x \to \infty} f(x + iy) = 0 \), uniformly for \( y \in [-\alpha, \alpha] \) for \( 0 < \alpha < 1 \).

2. Problem 18, Chapter 14: Let \( \Omega \) be an open simply connected domain and \( f : \Omega \to D, g : \Omega \to D \) one-to-one conformal maps onto the unit disk \( D \). What relationship exists between \( f \) and \( g \)? What if \( f(z_0) = g(z_0) = a \) for some \( z_0 \in \Omega, a \in D \)?

3. Problem 26. This is a problem outlining the proof due to Koebe of the Riemann mapping thm. Please note that Rudin denotes the unit disk by \( U \).

4. Problem 29. This is about the composition powers of a function \( f : \Omega \to \Omega, f \in H(\Omega) \), assuming \( f(a) = a \) for some \( a \in \Omega \).

5. Problem 32. This problem asks to compute the image of various sets under

\[
    f(z) = \exp \left\{ i \log \frac{1+z}{1-z} \right\}.
\]