CS 311	**BigIdeas**	**Homework 2**
Fall 2004		**Due: October 15, 2004**

1. The (abstract) syntax of MinML is

$$\begin{array}{rcl} e & ::= & n \mid \texttt{true} \mid \texttt{false} \mid x \mid \\ & & +(e_1, e_2) \mid *(e_1, e_2) \mid =(e_1, e_2) \mid <(e_1, e_2) \mid \\ & & \texttt{if}\ e\ \texttt{then}\ e_1\ \texttt{else}\ e_2 \mid \\ & & \texttt{apply}(e_1, e_2) \mid \texttt{fun}\,(x : \tau_1) : \tau_2\ \texttt{is}\ e \end{array}$$

where n ranges over integer constants and x ranges over a set of variables. In this language a *value* is either a number, a boolean constant, or a function expression. The dynamic semantics of evaluation is given by a transition relation defined by the rules

$$\frac{}{+(n_1, n_2) \mapsto m} \qquad \text{where } m = n_1 + n_2$$

$$\frac{e_1 \mapsto e_1'}{+(e_1, e_2) \mapsto +(e_1', e_2)} \qquad \frac{e_2 \mapsto e_2'}{+(v_1, e_2) \mapsto +(v_1, e_2')}$$

$$\frac{}{\texttt{if true then}\ e_1\ \texttt{else}\ e_2 \mapsto e_1} \qquad \frac{}{\texttt{if false then}\ e_1\ \texttt{else}\ e_2 \mapsto e_2}$$

$$\frac{e \mapsto e'}{\texttt{if}\ e\ \texttt{then}\ e_1\ \texttt{else}\ e_2 \mapsto \texttt{if}\ e'\ \texttt{then}\ e_1\ \texttt{else}\ e_2}$$

$$\frac{e_1 \mapsto e_1'}{\texttt{apply}(e_1, e_2) \mapsto \texttt{apply}(e_1', e_2)} \qquad \frac{e_2 \mapsto e_2'}{\texttt{apply}(v_1, e_2) \mapsto \texttt{apply}(v_1, e_2')}$$

$$\frac{}{\texttt{apply}(\texttt{fun}\,(x : \tau_1) : \tau_2\ \texttt{is}\ e, v) \mapsto \{v/x\}e}$$

Here we presented representative rules for the $+$ operator. Similar rules will cover the $*$ operator and the relational operators $=$ and $<$.

The typing rules for MinML are given below(omitting the rules for $*$-expressions and $=$ expressions, which are the same as for $+$ and $<$).

$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}$$

$$\frac{\Gamma \vdash e_1 : \texttt{int} \qquad \Gamma \vdash e_2 : \texttt{int}}{\Gamma \vdash +(e_1, e_2) : \texttt{int}}$$

$$\frac{\Gamma \vdash e_1 : \texttt{int} \qquad \Gamma \vdash e_2 : \texttt{int}}{\Gamma \vdash < (e_1, e_2) : \texttt{bool}}$$

$$\frac{\Gamma \vdash e : \texttt{bool} \; \Gamma \vdash e_1 : \tau \qquad \Gamma \vdash e_2 : \tau}{\Gamma \vdash \texttt{if } e \texttt{ then } e_1 \texttt{ else } e_2 : \tau}$$

$$\frac{\Gamma \vdash e_1 : \tau_2 \to \tau \qquad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash \texttt{apply}(e_1, e_2) : \tau}$$

$$\frac{\Gamma[x : \tau'] \vdash e : \tau}{\Gamma \vdash \texttt{fun}(x : \tau') : \tau \texttt{ is } e : \tau' \to \tau}$$

Problem 1. Prove the following *Substitution Lemma*.

Lemma [Substitution]: If $\Gamma[x : \tau'] \vdash e : \tau$ and $\Gamma \vdash e : \tau'$, then $\Gamma \vdash e'/xe : \tau$.

Problem 2. Prove the following Cannonical Forms Lemma.

Lemma [Cannonical Forms]:. Let v be a value (*i.e.* a number constant, a boolean constant, or a closed function expression), and assume that $\vdash v : \tau$.

1. If $\tau = \texttt{bool}$, then either $v = \texttt{true}$ or $v = \texttt{false}$.
2. If $\tau = \texttt{int}$, then $v = n$ for some natural number n.
3. If $\tau = \tau_1 \to \tau_2$ then $v = \texttt{fun}\,(x : \tau_1) : \tau_2 \texttt{ is } e$, for some x and e.