The Internal and External Algebraic Structure of Complexity Classes

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Abstract

The report forms a bibliographic companion to a talk given at the Workshop on Algebraic Methods in Complexity Theory in Madras, India in December, 1994. In this report we will give many references that the reader may find useful in studying the internal and external algebraic structure of complexity classes.

1 Introduction

The external algebraic structure of a complexity class refers to algebraic closure properties that a class may possess such as the fact that \#P functions are closed under addition and multiplication. The internal algebraic structure refers to how we can give alternate definitions of complexity classes based on building these classes from algebraic operations on top of very simple functions. For example \#P functions in some sense look like low-degree polynomials over their inputs.

This report will give a list of many references related to the internal and external algebraic structure of complexity classes. Please note that this report is in no sense complete and only meant as a place for interested researchers to find some references in this area.

2 Study of Structure

This section lists some papers that have studied algebraic structures of complexity classes in their own right.

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Papers on Study of Structure


3 External Structure

In this section we list papers that prove theorems using the external structure of complexity theory. These theorems generally show that classes have certain closure properties based on their external algebraic structure. We feel that this study may prove more important as it may lead us to understand how to separate complexity classes. If two complexity classes do not have the same external algebraic structure then they cannot coincide.

Papers Using External Structure


4 Internal Structure

In this section, we list papers that use the internal algebraic structure of complexity classes to prove theorems about these classes. By expressing a complexity class in terms of simple algebraic operations, we can often “simulate” that class using other means. Many important circuit complexity and interactive proof system results use this method.

Papers Using Internal Structure


5 Internal and External Structure

The papers that we find most interesting use both the internal and external structure of complexity classes. These papers use the internal structure to "simulate" the classes and the external structure to keep these operations within the class. This section includes some of the most important results in complexity theory.

Papers Using Both Internal and External Structure


