CMSC 23710/33710 **Winter 2013**	**Scientific Visualization**	**Homework 1** **Assigned Jan 8** **Due Noon Jan 15**

(Updated Jan 10: fixed typo in 2nd paragraph: "should by you" → "should be done by you")

The point of this homework is to (1) see the kind of mathematics that are assumed by the lectures and course materials, and (2) to make sure that you are able to **electronically** hand in assignments via SVN as described on the SVN for SciVis page http://people.cs.uchicago.edu/ glk/class/scivis/svn.html.

You are to complete this individually (as with all the homeworks in this class). For this assignment you do not need, and must not use, a calculator or a computer program (Matlab, Mathematica, Python, etc.) to compute the answers; only use a computer to write it up and hand it in. The work of answering these questions should be done by you, **in your head or "by hand"**. The questions test mathematical knowledge and insight of the sort assumed in this class, so you are doing yourself a disservice if you use a calculator or computing program. When the course description says that a prerequisite is "basic knowledge of linear algebra and calculus", and the course web page says "Your math background must include linear algebra and calculus", this is referring to things like:

- operations with matrices and vectors
- representing a vector relative to a given basis
- column-space and row-space of matrices
- the real $\mathbb{R}$ and complex $\mathbb{C}$ numbers
- limits, continuity, and differentiability of functions
- integrals of real-valued functions
- Taylor expansions of functions

This is what the questions below ask about. **If doing this assignment takes more than about two hours, or if you need to consult many web pages or textbooks to learn the material to find the answers, your math background may not be a good fit for this class, and this class may not be a good fit for you.**

There are enough questions, and the questions are simple enough, that we will not be giving partial credit for incorrect answers. For this homework, we are looking for short answers, not explanations (future assignments may require more explanatory writing). **All questions have equal weight**.

1. Let A, B, and C be invertible matrices. What is $(ABC)^T$, in terms of A^T, B^T, and C^T?

2. (same A, B, C as previous) What is $(ABC)^{-1}$, in terms of A^{-1}, B^{-1}, and C^{-1}?

3. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$. What is $A^T A$? (write out the 2×2 matrix of integers)

4. (same A as previous question) What is AA^T?

5. Are $A^T A$ and AA^T both always symmetric, for any square A? (yes or no)

6. Let $B = R \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} R^{-1}$ where R is some invertible 3×3 matrix. What is B^{-1}?

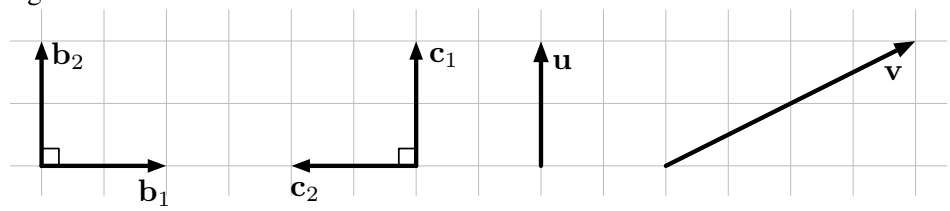
7. Let $u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $v = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$. What is $u^T v$?

8. (same u and v as previous) What is uv^T?

9. Let $u = \begin{bmatrix} -2 \\ 4 \\ 1 \end{bmatrix}$, $A = \begin{bmatrix} 3 & -10 & 6 \\ 2 & -1 & 4 \\ -2 & -16 & -4 \end{bmatrix}$, and $v = \begin{bmatrix} 4.78036 \\ 3.61452 \\ 2.93908 \end{bmatrix}$. What is $u^T A v$?

10. Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -2 & -1 \\ 1 & 1 & -2 \\ 0 & 0 & 5 \end{bmatrix}$, and $v = \begin{bmatrix} 6.31074 \\ 2.54163 \\ 8.00392 \end{bmatrix}$. What is ABv?

11. The following shows two bases $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ and two vectors $\mathbf{u}$ and $\mathbf{v}$, drawn to scale and on top of a regular grid.



The matrix representation of $\mathbf{x}$ in basis $\mathcal{B}$, notated $[\mathbf{x}]_{\mathcal{B}}$, is the column vector of coordinates $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ for which $\mathbf{x} = x_1\mathbf{b}_1 + x_2\mathbf{b}_2$. What is $[\mathbf{u} + \mathbf{v}]_{\mathcal{B}}$? (we're looking for a two integers, e.g. $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$)

12. (same set-up as above) What is $[\mathbf{u} - \mathbf{v}]_{\mathcal{B}}$?

13. (same set-up as above) What is $[\mathbf{u} + \mathbf{v}]_{\mathcal{C}}$?

14. (based on set-up above) Suppose there's a new basis $\mathcal{S} = \{\alpha\mathbf{b}_1, \beta\mathbf{b}_2\}$, $\alpha \neq 0$, $\beta \neq 0$; that is, a scaling of the $\mathbf{b}_i$ vectors in $\mathcal{B}$. If $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, what is $[\mathbf{x}]_{\mathcal{S}}$?

15. Define an equivalence relation $\sim$ on points (x, y) on the 2D Cartesian plane by

$$(x, y) \sim (u, v) \Leftrightarrow x^2 + y^2 = u^2 + v^2$$

In general, what is the geometric shape of the equivalence classes under $\sim$? (one word answer suffices)

16. A function $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x^n$ for some integer n. For what n is f continuous?

17. (same f as previous) For what n is f monotonic?

18. A function $f : \mathbb{R} \to \mathbb{R}$ is defined by

$$f(x) = \begin{cases} ax + b & x < 0 \\ \sin x & x \geq 0 \end{cases}$$

for some $a, b \in \mathbb{R}$. Give an example of specific numeric values for a and b that make f discontinuous. (answer in the form "$a = \quad, b = \quad$")

19. (same f as previous) Give values for a and b so that f is C^0 but not C^1 continuous (these are different orders of continuity).

20. (same f as previous) Give values for a and b so that f is C^1 continuous.

21. To a first-order approximation (in the sense of a Taylor series expansion around 0) what is $\sin 0.01$?

22. Let $f(x) = x^3 + \cos x$. To a first-order approximation (again in the Taylor series sense), what is $f''(\epsilon)$ for ϵ near 0?

23. Let $f(x) = 1 - |x|$. What is $\int_{-1}^{1} f(x)dx$?

24. Let $f(x) = \sqrt{1 - x^2}$. What is $\int_{-1}^{1} f(x)dx$? (hint: this is more about geometry than calculus)

25. What is

$$\lim_{n\to\infty} \sum_{m=0}^{n-1} \frac{m}{n^2}$$

where n is an integer? (hint: draw a picture in which $\frac{m}{n}$ and $\frac{1}{n}$ are the height and width of a rectangle)