CMSC 23710/33710	Scientific Visualization	Homework 3
Winter 2013		Assigned Feb 5
		Due Noon Feb 12

1. (40 points) This question concerns the index-to-world affine matrix transform. Let $\mathcal{B} = \{\mathbf{x}, \mathbf{y}\}$ be a basis for a two-dimensional world-space. Show your work.

(a) Let

$$M = \begin{bmatrix} \begin{bmatrix} \mathbf{u} \end{bmatrix}_{\mathcal{B}} & \begin{bmatrix} \mathbf{v} \end{bmatrix}_{\mathcal{B}} \end{bmatrix} = \begin{bmatrix} u_x & v_x \\ u_y & v_y \end{bmatrix}. \tag{1}$$

In terms of u_x, u_y, v_x, and v_y, what is M^{-1}?
(Remember Cramer's rule)

(b) Using the same M as above, let

$$T = \begin{bmatrix} \begin{bmatrix} \mathbf{u} \end{bmatrix}_{\mathcal{B}} & \begin{bmatrix} \mathbf{v} \end{bmatrix}_{\mathcal{B}} & \begin{bmatrix} \mathbf{z} \end{bmatrix}_{\mathcal{B}} \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$= \begin{bmatrix} M & z \\ 0 & 1 \end{bmatrix} \tag{3}$$

where $z = [\mathbf{z}]_{\mathcal{B}}$ is a column 2-vector, and "0" in (3) is a row 2-vector of 0s. Using the same short-hand for writing the 3×3 matrix T as a 2×2 matrix as in (3), what is T^{-1}, in terms of M and z?
(Multiply T by $S = \begin{bmatrix} N & w \\ 0 & 1 \end{bmatrix}$ and solve for N and w if $TS = \begin{bmatrix} I & 0 \\ 0 & 1 \end{bmatrix}$)

(c) Suppose image D is SI×SJ: the image values are $D[i, j]$ where $i \in [0, \ldots, \text{SI}-1]$ and $j \in [0, \ldots, \text{SJ}-1]$. With orientation transform T, image sample $D[i, j]$ has world-space coordinates $T \begin{bmatrix} i \\ j \\ 1 \end{bmatrix}$. Consider an SJ × SI image array E that is the transpose of D: $E[j, i] = D[i, j]$. What is the orientation transform T_E that permits E to appear the same as D in world-space (i.e. $T_E \begin{bmatrix} j \\ i \\ 1 \end{bmatrix} = T \begin{bmatrix} i \\ j \\ 1 \end{bmatrix}$)? Express T_E in terms of $[\mathbf{u}]_{\mathcal{B}}$, $[\mathbf{v}]_{\mathcal{B}}$, and $[\mathbf{z}]_{\mathcal{B}}$, similar to (2).

(d) Considering the same D as above, suppose an SI × SJ image array F is the result of flipping D along both axes: $F[\text{SI} - 1 - i, \text{SJ} - 1 - j] = D[i, j]$. What is the orientation transform T_F that permits F to appear the same as D in world-space? Express T_F in terms of $[\mathbf{u}]_{\mathcal{B}}$, $[\mathbf{v}]_{\mathcal{B}}$, $[\mathbf{z}]_{\mathcal{B}}$, SI, and SJ.

2. (30 points) You are given a function `show(V,R,C)` that displays 2-D arrays on screen as what look like matrices. The function takes a linearized 1-D list of values V (which can be indexed with $V[i]$, starting with $i = 0$), and two integer arguments R (number of rows) and C (number of columns). You also have a function `flip()`, which reverses a 1D list. How `show` interprets R or C as the size of a fast or slow axis, and how it arranges values on screen, and what conventions it does or doesn't obey, is not known. All we know is that `show` uses a self-consistent ordering to fill in rows and columns from the 1-D sequence of values in V: within each and every row, the columns are visited in the same sequence (either left-to-right or the reverse), and within each and every column, the rows are visited in the same sequence (either top-to-bottom or the reverse). **Unfortunately**, the behavior of `show` may be different between each of the sub-questions below. We are looking for short answers here, not long explanations.

(a) Here are two results from calling `show`:

$$\texttt{show(V,3,3)} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\texttt{show(flip(V),3,3)} \rightarrow \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

i. What might the underlying 1-D sequence of values in V be? There isn't a single correct answer.

ii. Is the row index fast or slow? Is column index fast or slow? Answer with "(row,col)=(fast,slow)" or "(row,col)=(slow,fast)".

(b) Two more results from `show` (with a different `V`):

$$\texttt{show(V,3,2)} \rightarrow \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$\texttt{show(V,2,3)} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

i. What is a possibility for the underlying 1-D sequence of values in V?

ii. Is this (row,col)=(fast,slow) or (row,col)=(slow,fast)?

(c) More results from `show` (with a different `V`):

$$\texttt{show(V,3,3)} \rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\texttt{V[0]} = 1; \texttt{V[2]} = 2; \texttt{V[6]} = 3; \texttt{show(V,3,3)} \rightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

i. Is this (row,col)=(fast,slow) or (row,col)=(slow,fast)?

ii. Let $\mathcal{B} = \{\mathbf{x}, \mathbf{y}\}$ be a basis for the screen space in which `show` displays. $\mathbf{x}$ points to the *right*, and $|\mathbf{x}|$ is the space between columns. $\mathbf{y}$ points *up*, and $|\mathbf{y}|$ is the space between rows. Disregard translation within the page. What 2×2 matrix M is `show` using to transform column 2-vectors $\begin{bmatrix} i \\ j \end{bmatrix}$ of index-space coordinates (with i *fast* and j *slow*) to world-space coordinates (in $\mathcal{B}$)?