

Updated Jan 9: fixed $\geq \rightarrow \leq$ interval definitions in last question.

The point of this homework is to (1) see the kind of mathematics that are assumed by the lectures and course materials, and (2) to make sure that you are able to **electronically** hand in assignments via SVN as described on the SVN for SciVis page <http://people.cs.uchicago.edu/~glk/class/scivis-2015/svn.html>.

You are to complete this individually (as with all the homeworks in this class). For this assignment you do not need, and must not use, a calculator or a computer program (Matlab, Mathematica, Python, etc.) to compute the answers; only use a computer to write it up and hand it in. The work of answering these questions should be done by you, **in your head or “by hand”**. The questions test mathematical knowledge and insight of the sort assumed in this class, so you are doing yourself a disservice if you use a calculator or computing program. The [course description](#) says that a prerequisite includes “basic linear algebra”, this is what the questions below ask about, as well as the basic Calculus reviewed on the first day of class. **If doing this assignment takes more than two or three hours, or if you need to consult many web pages or textbooks to learn the material to find the answers, your math background may not be a good fit for this class, and this class may not be a good fit for you.**

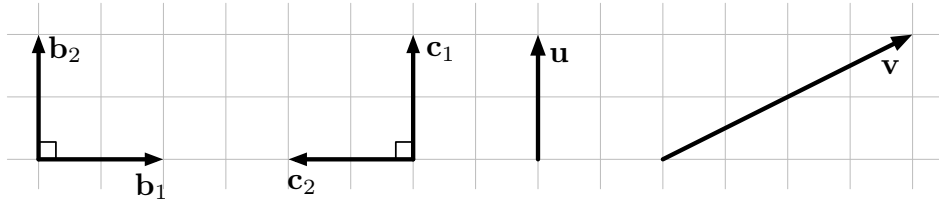
There are enough questions, and the questions are simple enough, that we will not be giving partial credit for incorrect answers. For this homework, we are looking for short answers, not explanations (future assignments may require more explanatory writing). **All questions have equal weight.**

1. Let A , B , and C be invertible matrices. What is $(ABC)^T$, in terms of A^T , B^T , and C^T ?
2. (same A , B , C as previous) What is $(ABC)^{-1}$, in terms of A^{-1} , B^{-1} , and C^{-1} ?
3. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$. What is $A^T A$? (write out the 2×2 matrix of integers)
4. (same A as previous question) What is AA^T ?
5. Are $A^T A$ and AA^T both always symmetric, for any square A ? (yes or no)
6. For symmetric and square matrices A and B , is AB always symmetric? (yes or no)
7. Let $B = R \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} R^{-1}$ where R is some invertible 3×3 matrix. What is B^{-1} ?
8. Let $u = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ and $v = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$. What is $u^T v$?
9. (same u and v as previous) What is uv^T ?
10. Define an equivalence relation \sim on points (x, y) on the 2D Cartesian plane by

$$(x, y) \sim (u, v) \Leftrightarrow x^2 + y^2 = u^2 + v^2$$

In general, what is the geometric shape of the equivalence classes under \sim ? (one word answer suffices)

11. The following shows two bases $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ and two vectors \mathbf{u} and \mathbf{v} , drawn to scale and on top of a regular grid.



The matrix representation of \mathbf{x} in basis \mathcal{B} , notated $[\mathbf{x}]_{\mathcal{B}}$, is the column vector of coordinates $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ for which

$\mathbf{x} = x_1 \mathbf{b}_1 + x_2 \mathbf{b}_2$. What is $[\mathbf{b}_2]_{\mathcal{B}}$? (we're looking for a two integers, e.g. $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$)

12. (same set-up as above) What is $[\mathbf{v}]_{\mathcal{C}}$?
13. (same set-up as above) what is $[\mathbf{u} + \mathbf{v}]_{\mathcal{B}}$?
14. (same set-up as above) What is $[\mathbf{u} - \mathbf{v}]_{\mathcal{B}}$?
15. (based on set-up above) Suppose there's a new basis $\mathcal{S} = \{\alpha \mathbf{b}_1, \beta \mathbf{b}_2\}$, $\alpha \neq 0, \beta \neq 0$; that is, a scaling of the \mathbf{b}_i vectors in \mathcal{B} . If $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, what is $[\mathbf{x}]_{\mathcal{S}}$?
16. A function $f : \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = x^n$ for some integer n . For what n is f continuous?
17. (same f as previous) For what n is f monotonic?
18. A function $f : \mathbb{R} \rightarrow \mathbb{R}$ is defined by

$$f(x) = \begin{cases} ax + b & x < 0 \\ (x + 1)^2 & x \geq 0 \end{cases}$$

for some $a, b \in \mathbb{R}$. Give an example of specific numeric values for a and b that make f discontinuous. (answer in the form " $a = \quad, b = \quad$ ")

19. (same f as previous) Give values for a and b so that f is C^0 but not C^1 continuous (these are different orders of continuity).
20. (same f as previous) Give values for a and b so that f is C^1 continuous.
21. What is the support of $f(x) = \max(0, 2 - x^2)$?
22. What is $\text{lerp}(0, 3, 11, 22, 44)$?
23. What is $\text{lerp}(-1, 1, -1, 0, 1)$?
24. Let $V[0] = 1, V[1] = 2, V[2] = 4, V[3] = 0, V[4] = 1$, and

$$k(x) = \begin{cases} 0 & x < -1 \\ x + 1 & -1 \leq x < 0 \\ 1 - x & 0 \leq x < 1 \\ 0 & 1 \leq x \end{cases}$$

What is $(V \otimes k)(1.5)$? Show your work.

25. Let $V[0] = 0, V[1] = 1, V[2] = 2, V[3] = 3, V[4] = 4$, and

$$k(x) = \begin{cases} 0 & x < -2 \\ x/4 + 1/2 & -2 \leq x < 0 \\ 1/2 - x/4 & 0 \leq x < 2 \\ 0 & 2 \leq x \end{cases}$$

What is $(V \otimes k)(2)$? Show your work.