An Algorithm for Moment-Based Global Registration of Echo Planar Diffusion-Weighted Images

G. Kindlmann1, A. L. Alexander2,3, M. Lazar2,3, J. Lee2, T. Tasdizen1, R. Whitaker1
1Computer Science, University of Utah, Salt Lake City, UT, United States, 2Medical Physics, University of Wisconsin, Madison, WI, United States, 3M. Lazar Lab, University of Wisconsin, Madison, WI, United States

Introduction
In a diffusion-weighted imaging experiment, the strong diffusion-sensitizing gradients can induce eddy currents, which will lead to image distortions in echo-planar images [1,2]. These distortions are typically represented with a 3-parameter model: scaling, translation, and shear along the phase-encoding direction. In diffusion-tensor MRI (DTI), these distortions will be different for each diffusion encoding direction and diffusion-weighting, leading to misregistration and errors in images calculated from two or more diffusion-weighted images (e.g., FA and trace(D)). A variety of methods have been proposed to correct or minimize the distortion effects including modifying the diffusion gradient waveforms [4], gradient amplifier pre-emphasis settings [6], distortion measurements in a phantom [5], and registration and modeling of the distortion [1, 3]. The latter approach is sensitive to the selection of a reference image for registration [1,5] and can be very complex if motion is also considered [3]. We have developed a novel fast and robust algorithm for the correction of eddy current distortions in diffusion-weighted images. The algorithm measures between-image distortions using low order moments of segmented diffusion weighted images, leading to a per-slice estimate of a linear model M of the imaging distortion. M maps from the diffusion-sensitizing gradient direction to the three parameters of the resulting eddy-current distortion.

Methods (Algorithm)
The algorithm can be summarized as: (1) Brain Segmentation: The brain is segmented from the background by thresholding and a combination of 2D and 3D connected components [7], creating a binary image mask. (2) Calculation of Moments and Transforms: Moments are a robust descriptor of object shape [8], calculated with $m_{ij} = \sum (x-x_i)^i (y-y_j)^j$ for (x,y) within the binary image mask of the brain. The scale S and shear H components of the transform can be recovered from $m_{20}$, $m_{11}$, and $m_{02}$ with $S = (m_{20}m_{12} - m_{10}m_{21}) / (m_{20}m_{12} - m_{10}m_{21})$ and $H = (m_{11} - m_{10}S) / m_{20}$, with primed moments computed from the target image. Translation $T$ is simply $(y') - (y)$. The transform $W_i$ (consisting of $H, S$ and $T$) from DWI $i$ to $j$ is calculated for all $(i,j)$ pairs. (3) Modeling Distortion due to Eddy Currents: Note that $W_i$ is equivalent to $W_jW_i^{-1}$, where $W_j$ is the transformation from a reference image to image $i$. The transformation $W_i$ due to eddy currents is modeled as a linear function of the diffusion-sensitizing gradient [1,3] via a 3x3 model matrix $M$: $[W_i] = [H \ 1 + S \ T]' = [M]G_i$, with $G_i$ and $M$ with one row per $(i,j)$ DWI pair. The process is repeated for each slice in the volume. (4) Warp Correction: By knowing the distortion model $M$, the transform $W_i$ caused by gradient $G_i$ can be determined from $M^*G_i$ and the correction of image $i$ is simply the inverse of $W_i$. The non-iterative nature of this algorithm contributes to its speed. No single step in this method is particularly compute-intensive, the slowest step currently is segmentation.

Results & Discussion
An example of the correction results for a 3T DTI study with 12 DWIs is shown in Figure 1. Results for a 40-slice volume took approximately 5 minutes to compute on a commodity PC. Because $W_i$ is the warp to DWI $i$ from the reference image without eddy current distortion (since $M^*0$ is the identity transform), the distortion correction maps the DWIs onto the T2-w $b=0$ image (Fig. 1d) without ever having used the T2-w image as a registration reference. The T2-w image is a poor reference for intensity based registration with the DWIs because of basic differences in contrast (e.g. CSF and asymmetric intensity variations due to anisotropy (white matter in DWI)). The algorithm has been applied to both 1.5T (Utah) and 3T (Wisconsin) DTI studies with different encoding sets and appears to be quite robust. The accuracy of the algorithm depends on the accuracy of the brain segmentation and binary mask, although the use of all DWIs to estimate $M$ imparts insensitivity to small errors in individual masks. Slice-to-slice consistency can be imposed by linear fitting of the distortion model $M$ across all slices [e.g. 3]. Although the algorithm presented here is based on image moments from a binary image mask, the same distortion modeling and unwarping methodology could also be adapted to intensity-based image registration methods.

References:
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Outline of Algorithm
1) Segmentation: In each DWI, the brain interior is segmented from the skull and background.
2) Transforms and Moments: Moments are calculated from segmented DWIs, from which the distortion transforms between all pairs of DWIs are determined.
3) Distortion Modeling: The mapping between the direction of the distortion-sensitizing gradient and the eddy current distortion is modeled as a 3x3 matrix.
4) Model Fitting: The previous steps are repeated on each slice of the image volume. Results may be improved at the top and bottom of the scan by fitting the model to a smooth variation across slices.
5) Distortion Correction: The distortion at each slice of each DWI is now known from the model. The DWIs are unwarpd and resampled onto a common grid.

3) Distortion Modeling
With the image moments and the formulas above, we can determine all pair-wise mappings from one distorted DWI to another.

But we need to recover the mapping of each distorted DWI back to the (undistorted) coordinates of the T2-weighted reference image R.

We accomplish this by modeling the relationship between the distortion model of each distorted DWI and the undistorted eddy current distortion. Our linear model has nine parameters:

\[ \begin{bmatrix} x' \\ y' \\ z' \\ x'' \\ y'' \\ z'' \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 & b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 & d_1 & d_2 & d_3 \\ e_1 & e_2 & e_3 & f_1 & f_2 & f_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \]

To assess whether the registration succeeded in mapping the DWIs to the undistorted space of the T2-weighted image, we divided the DWIs and the T2.-weighted reference image R.

4) Model fitting
Smaller, more complex shapes in slices at the top of the cortex, the auditory and visual areas, or the brainstem are problematic for segmentation, degrading registration results. The physical origin of the EPI distortion, however, suggests smooth variation with slice position, as observed by others. So that distortion estimation on some slices can improve estimation elsewhere, we quantify segmentation uncertainty on each slice in terms of a histogram of segmented DWI values at location \( x_i \), using their standard deviation, normalized by their L2 norm, summed over the image:

\[ \sum_i \sigma_v(x_i,y_i) \]

After sorting slices by segmentation uncertainty, a fraction of the most “certain” slices are used to determine a linear or parabolic parameter distortion model, as a function of slice position \( z \).

Future work will investigate higher-order fitting. The segmentation uncertainty can be inspected with stdv(\( x_i,y_i \)).

Discussion
The computational simplicity of computing moments, transforms, and models allows this method to be extremely fast. No iterative search or optimization is needed. Additional calibration or phantom scans are needed. In the current implementation, the skull/nonbrain is the DWI segmentation, not the registration itself.

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5) Distortion correction
We define the distortion model as a linear function of slice position \( z \). The EPI distortion in the DWI measured with gradient \( g \) is the \( [H_{g1},H_{g2}]z \) matrix found from \( \Delta g \).

Some distortion correction methods simply extrapolate along the phase-encoding direction; we use a high-quality filter, such as a sinc kernel with 10 sample support, to better preserve small image features. Internally is adjusted according to image scaling [8].

Results
The corrections are small, so directly inspecting the pre- and post-distortion images may not be informative. We inspect the performance of the model by comparing the histograms of the DWI values \( v(x,y) \), which is correlated with anisotropy, and which should be low in the gray matter, such as cortical surface.

This approximately the content of the poster presented by A. Alexander.

G. Kindlmann, A.L. Alexander, M. Lazar, J. Lee, T. Tasdizen, & R. Whitaker

1. Scientific Computing and Imaging Institute, University of Utah
2. Department of Medical Physics, University of Wisconsin-Madison
3. Utah Center for Advanced Imaging Research, University of Utah