Crease Features of Tensor Invariants

or,

Modeling White Matter Fiber Tracts Without Tractography

Gordon Kindlmann
Laboratory of Mathematics in Imaging
Department of Radiology
Brigham & Women’s Hospital
Harvard Medical School
gk@bwh.harvard.edu

Outline

- Background: DTI and fiber tracking
- Goal: Extracting white matter structure
- Invariants for measuring shape
- Method 1: Tensor field topology
- Method 2: Crease feature detection
- Combination: Fiber structures and boundaries
- Discussions & Ongoing Work
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Diffusion Weighted, Diffusion Tensor MRI

Single Tensor Model (Basser 1994) $A_i(b, g) = A_0 e^{-b g_i^T D g_i}$
Fiber Tractography

Integrate paths along $e_1$

Pierpaoli et al., 1997

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### Why “tracts without tractography”

**Standard pipeline:**

**DTI** → **10,000s of fibers** → **ROI analysis/clustering** → **anatomical regions**

- Want robust way of getting at major fiber structure
  - Less parameter tuning; closer to data
- Automatic landmarks for non-rigid registration
  - Fiber tract skeleton: “Sulci for white matter”
  - Enable group differences on tensor attributes
- Surgical planning of tumor resection
  - Measure tract deformation, asymmetry

### Registration challenge

<table>
<thead>
<tr>
<th>T2</th>
<th>FA</th>
<th>RGB($e_1$)</th>
</tr>
</thead>
</table>

- DTI shows major fiber orientation
- Don’t want to blur adjacent & orthogonal tracts
Background: DTI and fiber tracking

Goal: Extracting white matter structure

Invariants for measuring shape

Method 1: Tensor field topology

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Discussions & Ongoing Work

Space of Tensor Shape

\[ D = R \Lambda R^{-1} \]

\[ = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \]

\( \lambda_1 \geq \lambda_2 \geq \lambda_3 \)
Tensor shape parameterizations (Ennis & Kindlmann 2006)

\[ \text{tr}(\mathbf{D}) = D_{xx} + D_{yy} + D_{zz} \]

\[ |\mathbf{D}| = \sqrt{\text{tr}(\mathbf{D}^T \mathbf{D})} \]

\[ \mathbf{E} = \text{deviatoric}(\mathbf{D}) = \mathbf{D} - \text{trace}(\mathbf{D}) \mathbf{I} / 3 \]

Mode = \( \det(\mathbf{E} / |\mathbf{E}|) \) (Crisicone 2000)

Geometry of mode and eigenvalues

\[ \Theta = \cos^{-1}(\text{mode}) / 3 \]

\[ \lambda_1 = \frac{\text{tr}(\mathbf{D})}{3} + \sqrt{\frac{2}{3}} |\mathbf{E}| \cos(\Theta) \]

\[ \lambda_2 = \frac{\text{tr}(\mathbf{D})}{3} + \sqrt{\frac{2}{3}} |\mathbf{E}| \cos(\Theta - 2\pi / 3) \]

\[ \lambda_3 = \frac{\text{tr}(\mathbf{D})}{3} + \sqrt{\frac{2}{3}} |\mathbf{E}| \cos(\Theta + 2\pi / 3) \]
Software Demo

Human brain dataset (2x2x3 mm, 3T, 5 B0 + 30 DWI)

Purpose of visualizations: develop intuition for relationship between the spatial patterns of invariants and the underlying anatomy

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Method 1: Tensor Field Topology

- Vector Field Topology (Helman & Hesselink 1989)
  - Critical points: \( \mathbf{v} = 0 \)
  - Paths connecting critical points
  - Separatrices decompose field
- Applied to tensor fields (Delmarcelle & Hesselink 1994)
  - Degenerate points: 2 or 3 eigenvalues equal
  - Decompose field into eigenvector flows

Points where 2 eigenvalues equal

- Generically are lines (co-dimension 2, counting argument)
- Zheng et al. 2004, 2005
- Explicit root finding on

\[
D_3(T) = f_2(T)^2 + f_3^1(T)^2 + f_3^2(T)^2 + f_3^3(T)^2 + 15f_{z1}(T)^2 + 15f_{z2}(T)^2 + 15f_{z3}(T)^2
\]

Tensor Discriminant:
\[
D_3 = (\lambda_1 - \lambda_2)^2 (\lambda_1 - \lambda_3)^2 (\lambda_2 - \lambda_3)^2
\]

- Numerically find simultaneous roots of seven cubic polynomials
  
  Zheng 2004

Synthetic Elastic Stress Tensor
Degenerate Tensors in Real Data?

- Densely sampled discriminant in sagittal slice, at different scales ($\sigma$)

$\sigma = 1$  $\sigma = 2$  $\sigma = 3$

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Method 2: Ridge/Valley Detection

- Medial Axis Transform (Blum 1973)
  - For binary images
- Ridges and Valleys
  - For gray-scale images

Crease feature definition (Eberly 1994)

- Taylor expansion → Hessian
  \[ f(x_0 + d) \approx f(x_0) + d \cdot g(x_0) + d^T H(x_0) d / 2 \]
  \[ g = \nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix} \]
  \[ H = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial x \partial z} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} & \frac{\partial^2 f}{\partial y \partial z} \\ \frac{\partial^2 f}{\partial x \partial z} & \frac{\partial^2 f}{\partial y \partial z} & \frac{\partial^2 f}{\partial z^2} \end{bmatrix} \]

- Eigenvectors(H): 2nd-order structure orientation
- Extrema when \( g \) orthogonal to constraint surface
- Crease: Gradient orthogonal to one or two \( e_i \)
  - ridge surface: \( g \cdot e_3 = 0; \lambda_3 < \text{thresh} \)
  - valley line: \( g \cdot e_1 = 0; g \cdot e_2 = 0; \lambda_1, \lambda_2 < \text{thresh} \)
2D Ridge Line Example

<table>
<thead>
<tr>
<th>f</th>
<th>f &amp; isophotes</th>
<th>ridge line</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>g \cdot e_2</td>
<td>$</td>
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Basic Idea

Discriminant \((\lambda_1-\lambda_2)^2 (\lambda_1-\lambda_3)^2 (\lambda_2-\lambda_3)^2 = 0\)

\[\Rightarrow\] Two eigenvalues equal

\[\Rightarrow\] Mode at **global** extrema (-1 or 1)

- Generalize tensor field topology
- Crease features in tensor mode
- Crease features in FA

Crease features of tensor invariants

- Create smooth continuous tensor field
- 1st, 2nd derivatives of field (rank 3,4 tensors)
- Gradient, Hessian of invariant
- Initial results
Continuous tensor field: convolution of sampled coefficients with continuous reconstruction filters

\[ f[k] \ast w(x) = f(t) \]

To differentiate: convolve with derivative of reconstruction filter

\[ f[k] \ast w'(x) = f'(t) \]

Pajevic 2002

Measurement by convolution

Computing invariant derivatives

\[ FA = 3 \sqrt{\frac{Q}{S}} \]

\[ Q = \frac{S - J_2}{9} \]

\[ J_2 = D_{xx}D_{yy} + D_{xx}D_{zz} + D_{yy}D_{zz} - D_{xy}^2 - D_{xz}^2 - D_{yz}^2 \]

\[ S = D : D = \frac{D_{xx}^2 + D_{yy}^2 + D_{zz}^2}{4} + 2D_{xy}^2 + 2D_{xz}^2 + 2D_{yz}^2 \]

\[ \nabla J_2 = (D_{yy} + D_{zz}) \nabla D_{zz} + (D_{xx} + D_{zz}) \nabla D_{yy} + (D_{xx} + D_{yy}) \nabla D_{xx} \]

\[ -2D_{xy} \nabla D_{xy} - 2D_{xz} \nabla D_{xz} - 2D_{yz} \nabla D_{yz} \]

\[ \nabla Q = \frac{\nabla S - \nabla J_2}{9} \]

\[ \nabla S = 2D_{xx} \nabla D_{xx} + 2D_{yy} \nabla D_{yy} + 2D_{zz} \nabla D_{zz} \]

\[ + 4D_{xy} \nabla D_{xy} + 4D_{xz} \nabla D_{xz} + 4D_{yz} \nabla D_{yz} \]

\[ \nabla FA = \frac{3}{2} \left( \sqrt{\frac{1}{S Q}} \nabla Q - \sqrt{\frac{Q}{S^3}} \nabla S \right) \]

\[ \text{mode} = \frac{R}{\sqrt{Q^3}} \quad R = \frac{-5 \text{tr}(D) J_2 + 27 \det(D) + 2 \text{tr}(D) S}{54} \]

Hessian(FA), Hessian(mode) more involved

Why: Invariants and \( \nabla \) don’t commute; storage
Initial results

- \( -\lambda_3(H(FA)) \)
- \( 1 - |e_3(H(FA)) \cdot e_1(D)| \)

FA ridge surface, weighted

Initial results

- \( \text{RGB}(e_1) \)
- FA
Discussion & Ongoing Work

- Contributions
  - Extracting geometry from differential structure
  - Combining tensor topology & crease detection
- Scale space: interfaces versus cores
- Tensor eigensystem orientation
- Crease lines
- Evaluation on more datasets

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