Reading List for Murali Krishnan Ganapathy

**Notation:** $\mathcal{P}$ is a Markov Chain on state space $\mathcal{X}$, total variation mixing time $T$ with stationary distribution $\pi$. $\pi_* = \min_x \pi(x)$ and reverse chain $\mathcal{P}^{-1}(x, y) = \mathcal{P}(y, x)\pi(y)/\pi(x)$.

**Exclusion Process:** Given an $n$ vertex graph $G$ and $1 \leq d \leq n$, the configurations of the exclusion process are the locations of $d$ particles placed on the vertices of $G$. At each step a particle $v$ and neighboring location (determined by the graph) $w$ is chosen at random. If $w$ is not occupied by a particle, the particle at $v$ moves to $w$. The stationary distribution is the uniform distribution on all $\binom{n}{d}$ configurations. If the underlying graph is undirected the process is called *symmetric* and called *asymmetric* otherwise.

**Matchings Chain:** Given a bipartite graph $G = (L, R, E)$, Broder defined a Markov Chain on the union of perfect and near perfect matchings of $G$. Given the current matching $X$, we choose an edge $e$ of $G$ at random do the following:

- If $X$ is a perfect matching and $e \in X$, move to $X - e$.
- If $X$ is near-perfect and $X + e$ is a perfect matching, move to $X + e$.
- If $X$ is near-perfect and one end of $e$ is matched, move to $X + e - f$ where $f \in X$ has a common vertex with $e$.
- Stay at $X$ otherwise.

**Graph coloring chain:** Given a graph $G$ and $k$, the states of the chain are $k$-colorings of $G$ (assuming $k$ is large enough). At each step we pick a vertex $v$ at random and a color $c$ at random. If recoloring $v$ to $c$ results in a proper coloring we do it, else we stay at the current coloring.

**Two dimensional shuffle:** Start with $\ell \times m$ cards arranged on a grid. At each step pick a card at random and exchange it with one of its neighbors.

**Thorp Shuffle:** Suppose we have $2n$ cards. Split pack of cards into two groups containing the top $n$ cards and bottom $n$ cards respectively. For each $1 \leq i \leq n$ drop the $i$’th cards in the two groups in one of the two possible orders.


**Abstract:** Given a non-reversible Markov Chain $\mathcal{P}$, derive mixing time bounds for $\mathcal{P}$ in terms of the Dirichlet forms of the multiplicative symmetrization $\mathcal{P} \mathcal{P}^{-1}$ and additive symmetrization $(\mathcal{P} + \mathcal{P}^{-1})/2$ of $\mathcal{P}$ and use it to estimate mixing time of asymmetric exclusion process on the circle.

**Abstract:** Given a reversible $P$, estimate spectral gap of $P$ via the maximum congestion in a family of paths $\{\gamma_{xy}\}_{x,y \in X}$, where $\gamma_{xy}$ connects $x$ to $y$ using edges corresponding to non-zero entries of $P$. This improves the mixing time of the matchings chain.


**Abstract:** Given a reversible $P$, estimate spectral gap of $P$ via multi-commodity flows. Also for any reversible $P$ constructs multi-commodity flow to prove $P$ mixes in time $T^2 \log(1/\pi\ast)$ where $T$ is mixing time of chain. Thus a chain is rapidly mixing iff it has a low cost multi-commodity flow. Applications include estimating mixing time of matchings chain.


**Abstract:** Compare the mixing times of reversible random walks on two different Cayley graphs of the same group $G$, by writing generators of one chain in terms on the generators of the other chain. Applications include estimating mixing time of the “two dimensional shuffle” and the shuffle generated by transposition and the $n$-cycle.


**Abstract:** Generalizes previous paper to compare mixing times of two arbitrary reversible Markov Chains on the same state space but possibly different stationary distributions. Applications include estimating mixing time of symmetric exclusion process on an arbitrary graph.


**Abstract:** The Blanket time of a chain is the time taken for the observed frequency of visits to be at least $1/2$ the expected frequency for all vertices. Conjectures that the blanket time is only a constant factor more than the cover time (time to visit all states once). Establishes the conjecture for a class of graphs which includes random graphs, hypercube and grids of dimension $\geq 2$. Also establishes the conjecture for paths and cycles.

Abstract: Introduces the concept of path coupling. Given a integer valued bounded metric on the state space, enough to show that for any two states at distance 1 the expected distance after one coupling step falls. One can then conclude that the same holds for two arbitrary states. Applications include improved/simpler analysis for many chains including graph coloring chain.


Abstract: Considers the idea of speeding up a reversible Markov Chain by splitting one state into multiple states, so that the new chain is a “cover” of the old chain. Shows that the mixing time of old chain is at most quadratic in the mixing time of the new chain (modulo log 1/π∗ factor).


Abstract: Uses the idea of graph powering and taking Zig-Zag products to show that any undirected regular graph G can be transformed in log-space to an expander G′ which contains connectivity information of G. Hence it follows that checking whether an undirected graph is connected can be solved in log-space, proving L=SL.


Abstract: Morris introduced the idea of Evolving sets and used it to estimate mixing times of asymmetric exclusion process and thorp shuffle. This paper sharpens the argument behind evolving sets and shows how one can derive mixing time bounds in many natural measures (variation, uniform, L2, relative entropy).