Abstract Algebra Math 254 Challenge problem

1. Recall that in class we investigated the symmetries of simple geometric objects. In particular we saw that there are 8 symmetries of the square, 4 reflections and 4 rotations. Here a symmetry is a rigid motion that takes the square onto the square. (Note that we think of the reflections of the square as taking place in 3 dimensions even though the square sits in 2 dimensions.)

   Define the n dimensional cube C_n to be the collection of points $(x_1, x_2, \ldots x_n) \in \mathbb{R}^n$ such that $0 \leq x_i \leq 1$ for all i. Thus C_1 is a line segment. C_2 is the square. C_3 is the standard cube.

   Show that the n dimensional cube C_n has $2^n n!$ symmetries, thus giving a group of symmetries of order $2^n n!$.