Algorithms CMSC-37000 Second Quiz. February 7, 2008
Instructor: László Babai

Name: ________________________________

Show all your work. **Do not use book, notes, or scrap paper.** Write your answers in the space provided. You may continue on the reverse. When describing an algorithm in pseudocode, **explain the meaning of your variables** (in English). This quiz contributes 6% to your course grade.

1. (6 points) Recall that Jarník's (a.k.a. Prim's) algorithm grows a minimum cost spanning tree from one vertex. The pseudocode for this algorithm is almost identical to Dijkstra's. There is a difference in the way each algorithm updates. State those lines where a grey vertex $y \in \operatorname{adj}(x)$ is being updated (left column: Dijkstra, right column: Jarník; if lines identical, just write "(same)" in the right column). Begin with the line "**elseif** $s(y) =$ grey **then**"

2. (10+6B points) Let $\{a_n\}$ be a sequence of positive integers. Prove: (a) If $a_n! \gtrsim n$ then $a_n \gtrsim \ln n/\ln\ln n$. (b) BONUS: Prove that the converse is false.

3. (4+4+4 points) Decide which of the following statements are loop-invariants for Dijkstra's algorithm. Reason your answers. Assume the priority queue consists precisely of the grey vertices, with the right keys (we do not view the priority queue as a separate variable for the purposes of the definition of the configuration space). (a) All black vertices are accessible. (b) All accessible vertices are black. (c) All accessible vertices eventually become black.

4. (2+8B points) A divide-and-conquer algorithm reduces an instance of size n to n instances of size $n/2$ each. The cost of the reduction is $O(n)$. Let $T(n)$ denote the cost of the algorithm on the worst instance of size n. (a) State the recurrent inequality for $T(n)$ that follows from such a reduction. (b) BONUS: Ignore the cost of reduction. Show that $T(n) = O(n^{c \log n})$ for some constant $c > 0$. Determine all values of c for which this conclusion is correct. You may assume n is a power of 2.