Algorithms CMSC-37000 Third Quiz. March 11, 2008
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Name: ________________________________

Show all your work. **Do not use book, notes, or scrap paper.** Write your answers in the space provided. You may continue on the reverse. When describing an algorithm in pseudocode, **explain the meaning of your variables** (in English). This quiz contributes 6% to your course grade.

1. (4 points) Prove that given n, the n-th Fibonacci number F_n cannot be computed in polynomial time.

2. (4+1+4B points) **(a)** Write the shortest 3-CNF formula (3 distinct literals per clause) that is not satisfiable. Prove that your formula is not satifiable. **(b)** Use part (c) to prove that your formula is the shortest (has fewest clauses.) **(c)** BONUS. Prove: if a 3-CNF formula consists of m clauses then at least $7m/8$ of the clauses are simultaneously satisfiable.

3. (2+4 points) Students are asked to define what it means that the language $L_1 \subseteq \Sigma_1^*$ is Karp-reducible to the language $L_2 \subseteq \Sigma_2^*$. Jeremy writes this: "There exists a polynomial-time computable function $f : \Sigma_1^* \to \Sigma_2^*$ such that $(\forall x \in \Sigma_1^*)(x \in L_1 \Rightarrow f(x) \in L_2)$."
(a) Correct Jeremy's error. **(b)** Determine exactly which languages are reducible to $L_2 =$ 3-COL in Jeremy's sense. Prove your answer.

4. (2+2+5 points) **(a)** Define the language "KNAP" that corresponds to the decision version of the Knapsack problem. **(b)** Define the "SUBSET-SUM" language. **(c)** Prove that KNAP is NP-complete, assuming SUBSET-SUM is NP-complete.

5. (6 points) (Medians of k-windows) Let $k > 0$ be odd. Given an array $A[1..n]$ of reals, let $B[i]$ be the median of the "window" $A[i..i+k-1]$ (k numbers). Compute the array $B[1..n-k+1]$ in $O(n \log k)$. Use pseudocode to describe the overall cycle structure; refer to procedures discussed in class by their names without much detail.

6. (BONUS: 4B points) Prove: if 3-SAT can be recognized in $O(n^{\log n})$ then every language $L \in \text{NP}$ can be recognized in $n^{O(\log n)}$. Explain why the big-Oh migrated to the exponent and compute the implicit constant in the exponent as a function of a parameter involved in the $L \leq_{\text{Karp}}$ 3-SAT reduction.