

CMSC-37110 Discrete Mathematics
QUIZ October 8, 2008

Name (print): _____

Do not use book, notes, scratch paper. Show all your work. If you are not sure of the meaning of a problem, **ask the instructor.** The *bonus problems* are underrated, do not work on them until you are done with everything else. **Write your solution in the space provided.** You may continue on the reverse. This exam contributes 8% to your course grade.

All numbers in the problems below are integers.

1. (3 points) True or false (state, prove): $(\forall x, y)(\exists z)(\gcd(x, z) + \gcd(y, z) = \gcd(x + y, z))$.

2. (6 points) Let $a = 4k - 3$ and $b = 3k - 4$. Prove: if a and b are not relatively prime then their g.c.d. is 7.

3. (4+4 points) (a) Find x such that $6\mathbb{Z} \cap 8\mathbb{Z} = x\mathbb{Z}$ or prove that no such x exists. (b) Find y such that $6\mathbb{Z} \cup 8\mathbb{Z} = y\mathbb{Z}$ or prove that no such x exists.

4. (7 points) Prove: if $\text{g.c.d.}(a, 91) = 1$ then $a^{12} \equiv 1 \pmod{12}$. ($91 = 7 \cdot 13$.)

5. (7+3 points) (a) Find infinitely many solutions to the equation $\varphi(n) = n/3$. (b) (BONUS) Find all solutions; prove it's all.

6. (3B points) BONUS PROBLEM. Prove: if $n \equiv -1 \pmod{4}$ then n cannot be written as a sum of two squares.