1. (4 points) True or false (circle one, prove):
   \((\forall x, y)(\exists z)(\gcd(x, z) + \gcd(y, z) = \gcd(x + y, z))\).

2. (10 points) Prove: \(\ln(1 + 1/n) \sim 1/n\).

3. (6+14 points) Count the (a) increasing (b) nondecreasing functions
   \(f : [k] \to [n]\). (Recall the notation \([k] = \{1, \ldots, k\}\).) Your answers
   should be simple expressions involving binomial coefficients. Prove your
   answers.
4. **(7+7 points)** Give a closed-form expression (no summation symbols, no dot-dot-dots) for each sum: (a) $\sum_{k=0}^{n} (2/3)^k$; (b) $\sum_{k=0}^{n} \binom{n}{k} (2/3)^k$

5. **(12 points)** Find values $a, b, c$ such that the sequence $x_n$ defined by the recurrence $x_n = a + bx_{n-1}$ with initial value $x_0 = c$ does not have a limit (either finite or infinite). Indicate why your sequence has no limit.

6. **BONUS (2B points)** Prove:

$$\prod_{n/2 < p \leq n} p < 2^n.$$  

(The product is over all primes in the range $(n/2, n]$.)

7. **BONUS (3B points)** Prove: the product of any $k$ consecutive integers is divisible by $k!$. (Hint: one-line proof.)

8. **BONUS (6B points)** True or false? Prove your answer.

$$\left(1 + \frac{1}{n}\right)^{n^2} \sim e^n.$$