Algorithms CMSC-37000 Third Quiz. March 9, 2010

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Name: ______________________________

Show all your work. **Do not use book, notes, or scrap paper.** Write your answers in the space provided and **continue on the reverse** if necessary. When describing an algorithm in pseudocode, **explain the meaning of your variables** (in English). WARNING: The bonus problems are underrated. Do the ordinary problems first. – This quiz contributes 6% to your course grade.

1. (10 points) Let k-COL denote the set of k-colorable finite undirected graphs. Describe a Karp-reduction from 3-COL to 4-COL. State the two facts that need to hold for your function to be a Karp-reduction. Do not prove.

2. (8 points if both answers correct, 0 otherwise) Decide whether each of the following statements is a loop-invariant for BFS. **Circle** one answer to each. Do not prove.
   (A) Vertex #2 is white. **Yes** **No**
   (B) Vertex #2 is black. **Yes** **No**

3. (8 points) Let $L_1 \subseteq \Sigma_1^*$, $L_2 \subseteq \Sigma_2^*$, and $L_3 \subseteq \Sigma_3^*$ be languages. Let f_1 be a Karp reduction from L_1 to L_2 with exponent C_1 (i.e., f_1 can be computed in $O(n^{C_1})$), and f_2 a Karp-reduction from L_2 to L_3 with exponent C_2. Combine these to a Karp-reduction g from L_1 to L_3. What is the exponent C of g? Briefly reason your answer.

4. (12 points) Prove that Euclid's algorithm runs in polynomial time. Prove any lemma you need, do not refer to it "from class."

5. (6 points) Consider the following two conjectures:
(A) P $\neq$ NP   (B) NP $\neq$ co NP.
If you prove (A), the Clay Institute will award you a million dollar prize. Suppose you proved (B). Can you claim the prize? **YES**   **NO**. (Circle one.) Reason your answer.

6. (8+8 points) (a) Give a Karp-reduction from 3-COL to HALTING. (b) Prove: there is no Karp-reduction from HALTING to 3-COL.

7. (BONUS: 6B points) Prove that (F_n mod m) can be computed in polynomial time. Indicate the key ideas only; use no more than three lines.

8. (BONUS: 8B points) Prove: if 3-COL can be Cook-reduced to factoring integers then NP = coNP. State the main steps before you begin the detailed proofs.