1. (4+8+8 points) Let \( A = (a_{ij}) \) be random \( n \times n \) (0,1)-matrix: each entry is 0 or 1, chosen at random by independent unbiased coin flips. 
   (a) What is the size of the sample space? 
   (b) Calculate the expected value of \( \text{Per}(A) \) and (c) \( \text{Det}(A) \). Your answers should be very simple formulas (closed form expressions, no summation or dot-dot-dot). Prove your answers to (b) and (c).

2. (8 points) Find the eigenvalues and an eigenbasis of the \( J \) matrix (\( n \times n \) all-ones matrix: all entries are 1).

3. (3+9+12 points) Let \( X \) be a projective plane or order \( n \) (meaning: the lines have \( n + 1 \) points). Let \( A \) be the incidence matrix of \( X \) (i.e., the (0,1)-matrix of which the rows are the incidence vectors of the lines). 
   (a) State the dimensions of this matrix. 
   (b) Compute the matrix \( A^T A \). 
   (c) Find \( |\det(A)| \).
4. (8 points) Recall that an orthonormal representation of a graph $G = ([n], E)$ (according to László Lovász) assigns unit vectors $v_1, \ldots, v_n$ to the vertices such that if $i$ and $j$ are distinct non-adjacent vertices then $v_i$ and $v_j$ are perpendicular. Determine, which graphs have an orthonormal representation in $\mathbb{R}^2$. Your answer should be a very simple characterization in familiar terms of graph theory. Do not prove.

5. (BONUS, 6B points) Let $p_1, p_2, p_3, p_4$ be four points in the Galois plane $PG(2, q)$ in general position (no three on a line). Let $r_1, r_2, r_3, r_4$ be another four points in general position. A “collineation” is a permutation of the points of the plane that preserves lines. Prove: there exists a collineation that moves $p_i$ to $r_i$ for all $i$.

6. (BONUS, 6B points) Prove: the automorphism group of the Petersen graph is isomorphic to $S_5$. 