CMSC-37110 Discrete Mathematics
FIRST QUIZ October 8, 2010

Name (print): ____________________________________

Do **not** *use book, notes, scratch paper.* **Show all your work.** If you are not sure of the meaning of a problem, **ask the instructor.** The *bonus problems* are underrated, do not work on them until you are done with everything else. **Write your solution in the space provided.** You may CONTINUE ON THE REVERSE. This exam contributes 6% to your course grade.

All variables in the problems below are integers except where expressly stated otherwise.

1. (3 points) True or false: "if $x \mid a$ and $y \mid b$ then $x + y \mid a + b$." Prove your answer.

2. (10 points) Prove: if $a \equiv b \pmod{m}$ and $x \equiv y \pmod{m}$ then $ax \equiv by \pmod{m}$. You may use without proof basic properties of divisibility (state what you use) and the fact that congruence modulo m is a transitive relation.

3. (6 points) Find all integers z such that $z \mid z - 4$. Prove your answer.

4. (2+2+2+2 points) Let a_n, b_n be sequences of real numbers. Assume $a_n \sim b_n$. Circle the correct answer.

   (a) Does it follow that $a_n + 1 \sim b_n + 1$? **YES NO**

   (b) Does it follow that $a_{n+1} \sim b_{n+1}$? **YES NO**

   If your answer is "YES," just circle, do not prove. If the answer is "NO," (c) give a counterexample and (d) state a natural sufficient condition under which the answer becomes "YES." Do not prove.

5. (18 points)
   Prove: $(\forall a)(a^{13} \equiv a \pmod{65})$. *Hint.* $65 = 5 \cdot 13$.

6. (15 points) Decide whether or not the following system of simultaneous congruences has a solution. **YES** **NO**
   Circle and prove your answer.

   $5x \equiv -4 \pmod{21}$

   $3x \equiv -5 \pmod{14}$

   Hint. Split each congruence into a pair of congruences modulo prime numbers.

7. (BONUS PROBLEM: 8B points) Let p be an odd prime divisor of the number $a^2 + 1$. Prove: $p \equiv 1 \pmod 4$.

8. (BONUS PROBLEM: 12B points) Let p^k be a prime power divisor of $\binom{n}{k}$. Prove: $p^k \leq n$.