CMSC-37110 Discrete Mathematics
THIRD QUIZ November 10, 2010

Name (print): ______________________________

Do **not** *use book, notes, scratch paper.* **Show all your work.** If you are not sure of the meaning of a problem, **ask the instructor.** The *bonus problems* are underrated, do not work on them until you are done with everything else. **Write your solution in the space provided.** You may CONTINUE ON THE REVERSE. This test contributes 6% to your course grade.

1. (8 points) Draw a simple connected planar graph in two ways in the plane so that the resulting dual graphs are not isomorphic. Indicate a simple reason why they are not isomorphic. Make your graph small (few vertices, few edges). To make your drawings clear and unambiguous, do not draw the duals. (You may draw the graph combined with the dual on the reverse.)

2. (12 points) Prove: for almost all graphs G, $\chi(G) > (\omega(G))^{100}$. Here $\omega(G)$ is the clique number (size of largest clique).

3. (17+3 points) (a) Prove: if every vertex of a connected plane graph has degree 3, and every region has 5 or 6 sides, then the number of 5-sided regions is 12. (b) Name a familiar plane graph that satisfies the conditions of (a).

4. (10+10 points) (a) Write the Newtonian binomial coefficient $\binom{-1/2}{n}$ as a closed form expression using an ordinary binomial coeffient and common arithmetic. (b) Asymptotically evaluate your expression. Your answer should be a very simple formula of the form an^bc^n where a, b, c are constants. Determine a, b, c.

5. (BONUS, 4B+3B points) Prove: in a planar graph (a) the number of paths of length 2 is $O(n^2)$ (where n is the number of vertices); (b) the product of the degrees is less than 6^n.

6. (BONUS, 8B points) Let $A_1, \ldots, A_m \subseteq \Omega$ be events. Assume $(\forall i)(P(A_i) = 1/2)$ and $(\forall i \neq j)(P(A_i \cap A_j) \leq 1/5)$. Prove: $m \leq 6$.

7. (BONUS, 6B points) Prove: if the graph G contains no 4-cycles then $\chi(G) = O(\sqrt{n})$.